A Longitudinal Cohort Study Examining the Relationship between Working Memory and UK Primary School Curricular Mathematics

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Abstract

Mathematics is an important skill that is taught to all children in the UK in a structured manner from a very early age. The purpose of this thesis was to examine how working memory (Baddeley & Hitch, 1974a; Baddeley & Hitch, 1994) and UK curricular mathematics are related, if specific components of working memory were more impactful upon performance in mathematics than others, and if we can predict mathematics outcomes using working memory measures. With reference to the influence of working memory on overall curricular mathematics performance, a cohort of 70 children from two primary schools in the North West of England was tested annually from their Reception year (mean age 5yrs 1m) at school to Year Two (mean age 6yrs 11m). The study used a number of working memory tasks, a UK curricular mathematics test, and two Performance Measures. This allowed data to be analysed both in a cross-sectional manner and longitudinally (Chapter 5).

The thesis also differentiates UK curricular mathematics into four separable "strands", Number, Calculation, Measures, Shape and Space, and Problem Solving. These strands are described consistently throughout the UK mathematics curricular literature (DfEE, 1999; DfEE & QCA, 1999a; DfES, 2003a) and the cohort data was used to statistically analyse the relationships between working memory and each strand in turn using a correlational design in Chapters 6 to 9.

Results indicated that working memory is a robust predictor of overall mathematics performance (Chapter 5), and of the Calculation Strand (Chapter 7). This finding was demonstrated in both the cross-sectional analyses and also in the longitudinal regression analyses. Of the working memory measures a distinct pattern of association was revealed. In particular the data imply that there is a strong role for the central executive at each age range, but in Year One verbal short-term memory emerges as an important predictor variable.

Working memory also showed significant predictive influence over the remaining three curricular mathematics strands that were measured, particularly at the youngest age grouping, but working

memory was not found to be a robust longitudinal predictor of Number, Problem Solving or Measures, Shape and Space.

The overarching conclusion is that working memory, and in particular the central executive, may support the development of early curricular mathematical skills independent of the influence of age and Performance Measures. The practical and theoretical implications are considered.

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"People like me don't do things like this"

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Chapter One

Mathematics is a highly important discipline. It equips us with the fundamental skills of logic and reasoning, and helps to train the individual to deal with complex problems. There are two key points to be made about mathematics in relation to this thesis. Firstly, mathematics is frequently considered to be a subject that a child either has an aptitude for or doesn't. In fact, mathematics encompasses a wide variety of skills and concepts. These skills and concepts are largely interconnected and often provide building blocks for others. However, despite this, it is feasible that some children can easily manage some skills and concepts but have difficulty with others. Secondly, mathematics is a very broad term routinely used to describe all of the related skills and concepts, and that impacts upon how we teach, learn, and measure mathematical abilities.

1 Literature Review

This literature review will provide a very brief overview of mathematics which will be expanded upon in greater detail in the context of the UK curriculum in Chapter 3. It also puts forward a synopsis of the Baddeley and Hitch model of working memory alongside previous research that has examined the specific components of the working memory model.

1.1 Mathematics

Mathematical skills from an educational perspective are instilled in us from a very young age. In the UK compulsory formal early years schooling begins during the first term after a child's fifth birthday (Education Act 1996). However, school based nursery places are available from the age of three to most children, and most children are in Reception classes at school from the age of four. With the advent of extra benefits for working parents we also see children in private nurseries from a very young age. According to estimates by the Department for Education and

Skills 88-99%[1] of the three year old population in England took up a place in early years education (DfES, 2003b) and mathematics forms a fundamental part of the school curriculum even at this very early age.

The school curriculum makes great attempts to break down the structure of mathematical teaching and learning into smaller, bite sized components in order to facilitate a building block style learning environment for children. The curriculum also dictates that education providers supply breadth of learning by integrating mathematical thinking into other subjects across the curriculum. The mathematical curriculum will be discussed in greater detail later in this thesis (Chapter 3).

In teasing apart the cognitive processes that are considered to be involved in attainment and application of mathematical skills and concepts, one can quite feasibly assume that processes such as language skills, reading ability, attentional demands, memory, sequential ordering, processing speed, and spatial elements are all among the elements thought to be involved. Memory clearly plays an important part in children's ability to understand mathematical concepts and performance on mathematical tasks. As an example, if we put a simple mathematical equation such as 8 + 5 to a child, then the child must first recognise the numbers and operands by means of long-term memory retrieval, then comprehend the nature of the problem to be solved, they must then recall an appropriate strategy for solving the problem, focus attention, retain the original sum and attempt to complete the calculation. So, in thinking about the step-by-step processes involved, a relatively uncomplicated mathematical calculation like 8 + 5 now appears to be considerably more complex.

In order to understand the memory processes involved in mathematics learning it is important to consider some of the principles of mathematics. Looking at the competencies in "number", it is

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[1] Based on slight over estimations due to rounding up census data including non-resident UK children and double counting where a child was educated at more than one establishment.

known that number learning is not as straightforward as it is presumed it to be. A child must first learn the English number words and their correct order ("one, two, three "), as well as the related Arabic numbers and their correct sequence ("1, 2, 3,"). Then the child would need to understand the quantities associated with these number words and Arabic numbers, so "six" and "6" are both symbols that represent a grouping of any six concrete or abstract things. Furthermore they also have to learn how to transcode numbers from one form to another, as in transcoding "twenty one" into "21."

Just as essential is the development of an understanding of the structure of numbers. Children must learn to recognise that numbers can be decomposed into smaller numbers and conversely, combined to generate larger numbers. The most complex characteristic of the number system is its base-10 structure, that is, the basic sequence of numbers repeats in series of 10 (e.g., 1, 2, 3, 4, 10 is repeated 10 + 1, 10 + 2, that is, 11, 12). This conceptual understanding of the base-10 number system is important as it provides the building blocks for the proficiency in other domains to occur, for example, complex arithmetic. Thinking about counting, “1”, “2”, and “3” is not a difficult sequence to learn by rote, but understanding the basic rules that underlie counting is evidently much more complex.

Early mathematical skills have been shown to be verbally encoded, and most Asian languages have linguistic counting systems past ten (ten-one, ten-two etcetera) whereas English number naming words (eleven, twelve, thirteen, fifteen) deviate from the typical base-10 system demonstrated in the most Asian languages (D'Amico & Passolunghi, 1999). Moreover, word problems offer an intricate relationship between language and mathematics, and the language of mathematics is critical to the understanding of even the most basic word problems. Verbal retrieval for archived information is essential to learning typically over-learned number facts such as multiplication tables and basic addition and subtraction facts. Terms like “such as”, “all”, “some”, “neither” may be confusing when embedded in the grammatical complexity of word problems (Stock, Desoete, & Roeyers, 2009) and it is worthy of note that children with mathematics disabilities frequently have delays in their language development also (Khng & Lee, 2009).

The principles of one-one correspondence, stable order, and cardinality characterise the "how to count" rules, and in turn these give the skeletal foundation for children's developing counting competencies. Children also frequently make inductions about the basic characteristics of counting by observing standard counting behaviours. For instance, in the western speaking world, because reading is performed from left to right, many young children believe that you must count from left to right; right to left counting would be deemed as erroneous. Berch and colleagues (Berch, Foley, Hill, & Ryan, 1999) provided evidence to this effect, that children represent numerical magnitude and in the form of a left to right mental number line.

In typical daily life both simple and complex arithmetical calculations are undertaken without even realising or noticing that we are performing a mathematical operation. How long until the end of the lecture? How much change should I get from my shopping? How many tins of cat food will I need for a week's supply? With a 20% discount voucher, how much will those new shoes cost now? Given that mathematics is embedded so deeply into our everyday activities as adults, understanding the early relationships between mathematics and working memory in a school setting is an important field of study.

This introductory chapter continues with an overview of the Baddeley and Hitch model of working memory (Baddeley, 1986a; Baddeley & Hitch, 1974a) and a very brief insight into other theoretical models. The subsequent chapter will consider in more detail the relationships between working memory and mathematics competencies and development, and finally the research goals of the present thesis will be explained.

1.2 Brief overview of the Baddeley and Hitch Model of Working Memory

In considering the in depth cognitive processes involved in mathematics it is important to evaluate the theoretical model of working memory (Baddeley, 1986a; Baddeley & Hitch, 1974a), as this model is to be the umbrella framework under which the thesis is based.

The working memory model is a theoretical account of the short-term storage and manipulation of information in cognitive tasks. The model, first proposed in 1974 by Baddeley and Hitch, took the form of a tripartite system that comprised of a domain general controlling element and two domain specific slave systems. The model has been revised as and when new empirical evidence has come to the fore, and the model has also undergone a number of minor name changes to the components within, and these more adequately reflect the theoretical advances.

In its original inception the model consisted of a domain specific phonological loop (PL) which is a temporary verbal-acoustic storage and rehearsal system. This was thought to be necessary in order to store and manipulate information of a verbal or acoustic nature, such as the retention and immediate recall of a list of digits, words or sentences. Latterly the phonological loop has been referred to as phonological or verbal short-term memory (V-STM).

The visuospatial sketchpad (VSSP) is a domain specific parallel slave system devoted to information and codes of a visual and spatial origin. It is thought to be used for temporary storage and manipulation of spatial and visual information, such as remembering shapes and colours, or the location or trajectory of objects/figures in space. It is also thought to be involved in tasks which involve planning of spatial movements, such as mapping a route through a maze. This slave system, like the phonological loop, also has a new identity. In more recent work the VSSP has been referred to as visual short-term memory (V-STM) or nonverbal short-term memory (NV-STM). In order to avoid confusion between the acronyms, this thesis will use the term nonverbal short-term memory (NV-STM).

The third element of working memory is purported to be the controlling aspect, this was initially termed the central executive (CE), and sometimes it is referred to as working memory. This thesis will use the term central executive -complex working memory (CE-CWM), although when referring to previous research where the term central executive was used the thesis will retain the term given by the authors. This element of working memory is considered to be a domain general limited capacity system. It is thought to have a number of distinct functions, including inhibition, task switching and monitoring the processing of temporarily held information, but may not be constrained to just those functions (Baddeley & Logie, 1999).

In terms of the functional limitations of central executive, it had initially been proposed that central executive span was limited by central executive capacity alone (Baddeley, 1966; Baddeley & Logie, 1999). However, a broad consensus has since been discussed that there is in fact no single factor constraining central executive (Conlin, Gathercole, & Adams, 2005; Miyake & Shah, 1999; Towse & Houston-Price, 2001). To this end it has been suggested by Barrouillet and Camos (2001) that both time and limitation of attentional resources constrain performance in working memory tasks. Bayliss, Jarrold, Gunn, and Baddeley (2003) substantiate the issue by arguing that children's spans on working memory tasks are underpinned by domain-general processing as well as domain-specific storage resources. Furthermore a wealth of research suggests that the processing element of verbal central executive tasks is supported by the central executive, but storage is managed by verbal short-term memory (Baddeley & Logie, 1999; Gathercole & Baddeley, 1993a, 1993b).

A substantial revision to the working memory model has incorporated a fourth component, the episodic buffer (Baddeley, 2000). This buffer is thought to be responsible for the assimilation of information from the subcomponents of working memory and subsequent integration with long-term memory. This thesis will not be making great consideration of the episodic buffer, primarily due to the significant methodological issues with tests that claim to measure this component. At the time of testing there were no episodic buffer tasks suitable for use with young children, and those tests available for use with adults were in their conceptual infancy.

1.3 Alternative perspectives on working memory

As with most theoretical constructs there are opposing and complimentary viewpoints offered by other researchers (for a very comprehensive overview of discussion in this area see Miyake & Shah, 1999). In terms of the alternative perspectives the theorists can be broadly split into two main camps, non-unitary as with the Baddeley model, and unitary theorists. In the cognitive sense, unitary refers to a cognitive model encompassing all the processes in one unit, whereas non-unitary refers to the model having separable elements within. The Baddeley and Hitch model is non-unitary in nature as the composition of the model can be divided into quite distinct separable components as and when empirically justified.

Nelson Cowan's Embedded Processes Model of working memory (Cowan, 1995, 1998) is of a unitary format, with the understanding that working memory and long term memory are not separable systems. However Cowan does acknowledge that this model is less unitary than some others as it encompasses activation outside attention (Cowan, 1995, 1998). Engle also purports a unitary model of working memory, but he takes this further and suggests that it is also domain free (2002). The primary difference in opinion over the non-unitary versus the unitary theories arise from a fundamental conceptual view of working memory as a domain-specific construct as opposed to a more general "whole" all-encompassing resource, however frequently the theorists do concur on other core concepts. Both sides of the working memory fence agree on the principle of working memory as a temporary storage and processing construct; that there are limited resources available for storage and processing within the construct; and that working memory is a current and active link with (or part thereof) long-term memory.

The remainder of this chapter will explore in greater detail the three core theoretical components of the Baddeley and Hitch working memory model and introduce the reader to the core concepts of working memory that this thesis will later call upon. The theoretical construct of working memory has been reviewed many times in the available literature. So for very comprehensive reviews of working memory, the key components and the use of working memory models in research see Gathercole (1999), Miyake and Shah (1999) and Baddeley (2001).

1.4 Evidence for the Working Memory Model

Alan Baddeley and Graham Hitch first proposed the working memory model in 1974 and the general concept of working memory was described by Baddeley (at a time when the model was being further developed) as encompassing "temporary storage of information that is being processed in any range of cognitive tasks" (Baddeley, 1986a).

The Baddeley and Hitch model has since seen several theoretical revisions and re-naming conventions, and the substantial addition in the form of the episodic buffer. A paper from 2001 by Alan Baddeley documented the historical background to the model, the empirical research that it is founded upon and its relevance in the current research environment. In the early days Baddeley and Hitch (1974a) postulated that the system comprised of a core element that is responsible for controlling two slave or sub-systems. They hypothesised that the sub-systems would temporarily allow the central executive to offload some of its short-term storage functions thus allowing the central executive to be free to proceed with the more complex aspects of processing (see Fig 1.).

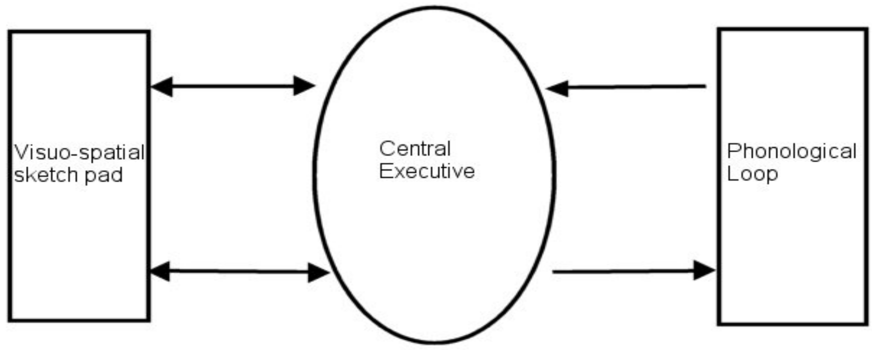

Figure 1. A simplified schematic representation of the working memory model (Baddeley and Hitch, 1974)

1.5 Components of the Working Memory Model

1.5.1 Phonological Loop - Verbal Short-Term Memory (V-STM)

Verbal short-term memory (V-STM) is considered to be responsible for the temporary storage of verbal and acoustic information. V-STM is theorised to consist of a passive, limited capacity phonological store that will retain information for approximately two seconds, coupled with an articulatory control process. In adults this process “refreshes” items to be recalled in the store by means of subvocal rehearsal (Baddeley, Lewis, & Vallar, 1984).

Current research hypothesises that the model has developed considerably in recent years and may now resemble Fig. 2 more closely.

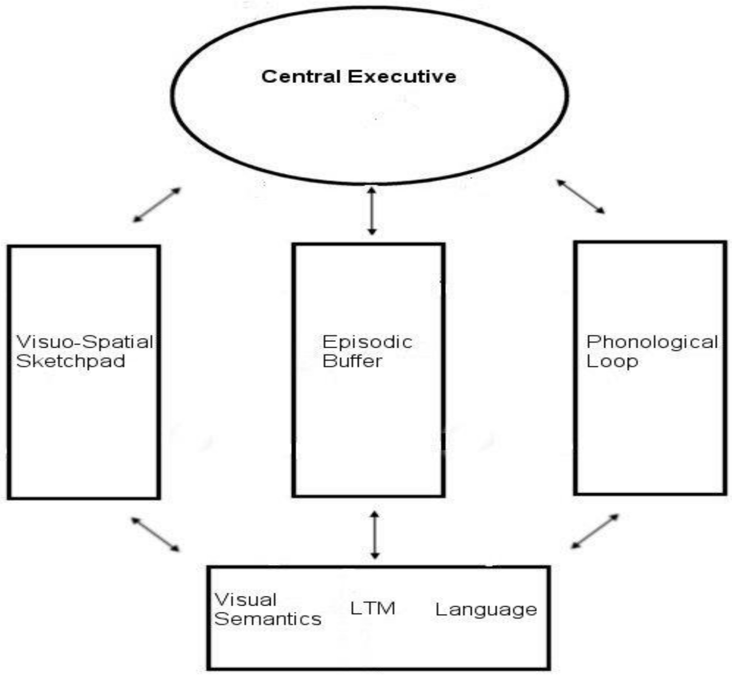

Figure 2. A revised schematic diagram of the working memory model (Baddeley, 2003)

Evidence for the phonological loop hypothesis arose from a mass of converging sources, considering aspects such as word length effect (Baddeley, Thomson, & Buchanan, 1975; Ellis & Hennelly, 1980; Johnston, Johnson, & Gray, 1987), phonological similarity effect (Conrad & Hull, 1964) as well as articulatory suppression (Richardson & Baddeley, 1975; Wilding & Mohindra, 1980). Some of the early key threads of empirical work allowed Baddeley and Hitch to decide that this subsystem of working memory was devoted to information of a verbal or auditory nature.

The passive phonological store is thought to be age invariant, and some studies with children have supported this (Hitch & Halliday, 1983; Hulme, Thomson, Muir, & Lawrence, 1984) with findings indicating age related increases in articulation rate (processing speed) rather than increases in the capacity of working memory. Hulme and colleagues (1984) demonstrated that from the age of 4 to adulthood memory span is an age invariant linear function of articulation rate insofar as the rate of articulation gains speed with age, then a proportional increase in memory span is observed, and the typical four year old child has a verbal short-term memory span that is about a $^{1}/_{3}$ of that of a typical adult (Pickering, Gathercole, & Peaker, 1998).

Subvocal rehearsal is a feature of V-STM whereby the individual will spontaneously rehearse to-be-recalled material for more effective recall. Typically this is evident by lip movements, or latterly (and more sophisticatedly) detected by movements in the throat (Fischer, 2008). Up to 7 years of age children typically do not seem to engage in subvocal rehearsal to assist active maintenance of a memory sequence (Flavell, Beach, & Chinsky, 1966; Gathercole, Adams, & Hitch, 1994) even though the cognitive architecture appears to be in situ (Johnston et al., 1987).

Gathercole and colleagues (1994; Gathercole & Hitch, 1993) have established that subvocal rehearsal is evident after the age of about 7 years old. This characteristic of working memory has been discussed as being an important factor in protecting memory traces from decay (Gathercole, 1998; Henry & Millar, 1993) as well as supporting the maintenance of accurate mathematical calculations (Logie, Gilhooly, & Wynn, 1994).

1.5.2 Visuospatial Sketchpad - Non-Verbal Short Term Memory (NV-STM)

As previously discussed, NV-STM maintains visual nonverbal information in short-term memory. It is thought that nonverbal short-term memory would be particularly specialized for tasks involving generation and manipulation of mental images, under the control of the central executive (CE-CWM).

In the original model it is apparent that far less consideration was given to the concept of “visual memory” as described in the 1974 book chapter (Baddeley & Hitch), and as such historically the nonverbal short-term memory system has been a challenge for researchers. This was largely due to theoretical considerations such as the exact nature of what kind of content that nonverbal short-term memory might temporarily store. Examples might include manipulation of mental imagery, visual information such as colour and object location, and also memory for movement (Quinn, 1994; Smyth & Scholey, 1994).

1.5.2.1 Development, Fractionation, dissociations and double dissociations in NV-STM

There have been challenges levelled at the assumption that nonverbal short-term memory is unitary in structure (see Mendez, 2001), however Robert Logie and his colleagues had, some time ago, already put forward an argument for fractionation of nonverbal short-term memory in the form of a cognitive model. They argue that the system comprises two separable elements, the inner scribe and the visual cache (Logie & Pearson, 1997; Salway & Logie, 1995). They proposed that these two elements would work in partnership with one another but under the fractionated view would have dissociable responsibilities. The inner scribe would hold information about movement sequences, spatial information, and would be linked to the planning and execution of movement, it is analogous to the "inner voice" or subvocal rehearsal idea from phonological research, whereas the visual cache would retain information primarily of visual form and colour.

Some empirical studies with child participants have facilitated theoretical support for this dissociation. In particular Logie and Pearson (1997) investigated the separability of visual and spatial working memory in children across a number of age ranges by administering a visual patterns task and a Corsi block task. They found that there was a much steeper age –related increase in scores on the VPT in comparison with the Corsi block task, the inference being that the visual element of nonverbal short-term memory is separate from the spatial element. Logie and Pearson present their work as evidence of fractionation of NV-STM, and that such dissociations are occurring in both adults and children (Smyth & Scholey, 1994; Wilson, Reinink, Weidman, & Brooks, 1968), so this is not merely a feature of changes during cognitive development. However, in a study that could show a potential confound in the Logie research Pickering and colleagues' study (2001) found similar developmental trajectories on NV-STM span tasks to those in these preceding studies, with the conclusion that if the developmental trajectories for the visual and spatial tasks were markedly different there would be evidence of developmental fractionation. In order to test the dissociation between static and dynamic presentation modalities Pickering et al chose to use a maze task so that they could illustrate the route out of the maze in both a static fashion (already drawn on to the maze) or dynamically, by means of tracing out the route with the finger . They also adopted the two presentation strategies for the visual patterns test allowing them to delineate between static and dynamic versions of both visual and spatial tasks. The anticipated trajectories of development growth in each task were obtained for both tasks (as in Logie & Pearson, 1997), however, rather than showing any developmental dissociation between visual and spatial working memory processes, the dissociation was between the static and dynamic versions of each of the two tasks. The researchers summarised that these findings provide complimentary evidence for developmental fractionation of the visuospatial memory system due to the belief that the nonverbal short-term memory subsystems are sensitive to task presentation format and the developmental fractionation appears to depend on the dynamic or static presentation mode more so than the type of information (visual or spatial) presented.

1.5.2.2 Nonverbal Short-Term Memory Capacity Limitations

There is some conflicting evidence and opinion regarding the functional capacity of the NV-STM subsystem. Luck and Vogel (1997) claim that NV-STM is severely limited in capacity, Logie (1995) suggested that it allows retention of visual and spatial items for a few seconds, furthermore Washburn and Astur (1998) failed to find any evidence of rehearsal in the visuospatial sketchpad, using a delayed matching-to-sample task. The table below (Table 1) summarises some capacity suggestions based upon research from a number of authors with differing perspectives.

Table 1. Examples of differing accounts of non-verbal short-term memory capacity

Author/s	**Date**	**Reported NV-STM Capacity**	**Participant Age or Age Range**
Luck and Vogel; Vogel, Woodman & Luck	(1997) (2001)	4 (+/- 1)	1997: Adults 2001: Adults
Cowan	(2001)	4 (+/-1)	Review paper
Phillips & Christie	(1977)	Capacity of 1 pattern	Adults
Alvarez & Cavanagh	(2004)	No more than 5 based on the equation: (amount of information/features)* (No items) = capacity	Adults
Song & Jiang (Shape)	(2006)	Depends upon: number of objects * visual information	Adults
Alloway, Gathercole & Pickering	(2006)	Span tasks Dot Matrix (mean): 14.99 Mazes Memory (mean): 9.32 Block Recall (mean): 14.37	4yrs-11yrs
Gathercole & Pickering	(2000a)	Static Mazes (means): 7.94 (f), 7.29 (m) Dynamic Mazes (means) 8.35 (f), 8.71 (m) Static Matrices (means): 16.35 (f), 17.26 (m) Dynamic Mazes (means): 9.69 (f), 8.88 (m)	7yrs 4mths

Clearly there are some discrepancies in the number of items/shapes/objects that the NV-STM system is believed to be capable of storing temporally. The structural evaluation of working memory in children that was completed by Alloway, Gathercole, and Pickering (2006) gives evidence that dependent upon the NV-STM task used, between the ages of 4 and 6 the mean score should be around 9 items (on the Mazes Memory task) and 14 items on the Block Recall task). Of further relevance may be Pickering et al (2001) which reports that observation of behaviour during the experiments suggested that older children demonstrated a tendency to phonologically recode visuospatial images into verbal codes. Examples they use are that children appeared to attribute a verbal label, such as a cross or a letter C, to the abstract visual patterns in the test. Hitch and colleagues (Hitch, Halliday, & Littler, 1989) argue that younger children rely on visual codes spontaneously, whereas older children show tendencies to rely more strongly upon verbal codes for pictorial material thus phonologically recoding the information. Moreover Miles and Morgan (1996) also noticed the occurrence of verbal recoding for visuospatial information. In an articulatory suppression task, performance on visuospatial tasks in 7 yr. olds and adults was significantly impaired, thus deducing that verbal strategies are supporting visuospatial recall. However they too found that there were no significant decrements in performance in younger children, surmising that this may be a feature related to age and development of phonological recoding in children's articulatory suppression. In addition to disrupting visuospatial recall using articulatory suppression there are a number of other methods utilised. Brief presentation of the stimuli reduces the ability of the participant to encode the stimuli phonologically (Frick, 1988). Another method to reduce verbal encoding of visual stimuli is to use items that do not have regular simple verbal codes, such as irregular shapes (Cermak & Levine, 1971) and it has also been shown that matrices appear resistant to verbal encoding (Phillips, 1974). Regardless, a definitive agreed capacity remains elusive.

1.5.3 Central Executive - Working Memory (CE-CWM)

Baddeley has described the hypothesised central executive as something of a "homunculus" or a "ragbag" (Baddeley, 1996a) , and its role in the model of working memory has been a topic that has not been without difficulty and argument. Quite what CE-CWM is and what CE-CWM does remains somewhat debatable, but the disentangling has begun in earnest.

1.5.3.1 Approaches to understanding CE-CWM

As mentioned earlier in this chapter, it is understood that the CW-CWM performs a variety of functions, such as co-ordinating the subsidiary systems, acting as an attentional controller, selecting certain streams of incoming information and rejecting others; task and strategy shifting; and selecting and manipulating information in long-term memory (Baddeley, 1996a). Examples of complex cognitive tasks where these functions may be applied could be situations or tasks that need planning or decision making, situations where responses are not well-learned or are likely to contain novel sequences of actions, and where suppression of irrelevant information is necessary to prevent cognitive overload, among others. Some level of dispute also arises in determining if there is a clear distinction between central executive and executive function. Executive functions are typically defined in clinical circles as higher level cognitive abilities that modulate initiation of behaviour, self-regulation, planning, and organisation (Lezak, 1983). However, in working memory theoretical models the central executive is also defined as utilising self-regulation, planning and organisation in terms of strategy selection, task switching, and controlling and inhibiting irrelevant information in attention. Teasing this apart or making specific definitions about the processes involved in central executive is a difficult task and Bull and colleagues (Bull, Espy, & Wiebe, 2008) have been working on this issue, and have some interesting findings, in particular pertaining to mathematics which will be discussed in greater detail in Chapter 2. However, with regard to a differentiation between executive function and central executive Bull et al use three measures that tap into executive function. Executive function measures are defined by the idea that unlike working memory measures, executive functioning tasks do not directly index the component skills needed for mathematics or reading. Therefore, this supposes that a lack of aptitude in such tasks is not due to a lack of knowledge relevant to the assessment domains, but because they are unable to inhibit, flexibly shift, and hold and manipulate information in short-term or working memory. There is however a degree of correspondence between executive function tasks and central executive-complex working memory tasks.

As with specifying nonverbal short-term memory, fractionation of the central executive system has been a consistently hot topic in the last decade. Fractionation of central executive (providing domain-specific processes to the sub-systems) has been proposed by several authors (see Goldman-Rakic, 2000; Shah & Miyake, 1996) and while the interpretations of such notions differ, a common thread appears to be that the central executive has a capacity devoted to linguistic/verbal storage and processing, and visual storage and processing.

Approaches to examining CE-CWM have differed somewhat. Two primary approaches have taken somewhat diverging methods to understanding this complex concept. Neuropsychologists have adopted a clinical, neuroanatomical approach, ordinarily based on evidence of frontal lobe damage in patients presenting with executive type disorders (Shallice & Burgess, 1991). This and other studies of a similar nature provide some functional anatomical evidence that the central executive is located in the frontal lobe region (Allain, Etcharry-Bouyx, & Le Gall, 2001; Collette & Van der Linden, 2002). The second key approach, and the one used in this thesis, is the psychometric approach. This has been strongly influenced by the theory provided by Baddeley and Hitch (1974) that working memory involves the concomitant storage and manipulation of to-be-recalled information as tasks have been created to measure these hypotheses.

Baddeley (1996b) describes his approach to studying the homunculus as being rather gradualist. This modus operandi classically acknowledges the homunculus with all its limitations, but it reasonably argues that such a concept is not only constructive in defining the extent of our attempts to understand the subsidiary slave systems, but could also be fruitful, provided there is a systematic attempt to analyse the functions performed by the homunculus. Given this premise there was a notion that if a series of executive processes could be identified and analysed, then the discipline could be better placed to understand if they were to be conceptualised as individual and separable functions, or whether a unitary account could be more appropriate. Overall, it may sound like a slightly disorganised approach to examining a long standing theoretical construct, but it does leave space for taking on board new ideas and allowing the model to develop as the research moves along. It also permits the previous research based upon the functional anatomical practices, and the psychometric methodologies to be considered, measured and contribute influence.

1.5.3.2 Measuring CE-CWM

CE-CWM measurement is typically characterised by a number of tests with paradigms that require simultaneous processing and storage demands. One of the early examples of a central executive task is Daneman and Carpenter's (1980) reading span test whereby the participants had to read a series of sentences aloud at their own pace and recall the last word of each sentence. The example that Daneman and Carpenter cite in their paper is "When at last his eyes opened, there was no gleam of triumph, no shade of anger. The taxi turned up Michigan Avenue where they had a clear view of the lake" (p453) and the correct recall items would be "anger", "lake". This test was devised using a series of cards, with a blank card indicating the end of each span set and the participant was previously advised that sets would increase in size, beginning with two sentences and two words to recall, up to six sentences with six final words to recall. Participants were given progressively longer sets of sentences until they failed all three sets at a particular level. The span test comprised three sets each of two, three, four, five, and six sentences with the span score being derived by calculating when a participant had attained at least two out of the three correct final word recalls from the set. The cognitive requirements of this task are clear in that it demands inhibition of irrelevant information (processing) and storage/rehearsal of the to-be-recalled words (Lobley, Baddeley, & Gathercole, 2005).

Daneman and Carpenter (1980) also introduce a listening span test which was a modified version of the reading span test. After hearing a sentence a verification response of true or false was required, and then at a signal the recall of the final words from each sentence was required. Given that the veracity of two (or more) statements required judgement while the last-word recall was an on-going process during the verification task, one expects the participant to be performing multiple cognitive actions whilst engaging with the task. As with the previous test the set sizes increased and spans also were derived similarly. One of the potential benefits of this test versus the reading test is that articulation rate can be maintained at a constant rather than being subject to the reading/articulation rate of the participant. In this particular study Daneman and Carpenter found that performance on both tasks predicted prose comprehension in college students, with reading span being marginally better than listening recall. Similarly Oakhill, Yuill and Parkin (1986) noted that when measuring working memory in younger children, those that were adept at reading but had poor comprehension skills also presented with a lower working memory span. Oakhill et al (1986) suggest that this is a feature of a deficit in the central executive and this is largely because in order to manage prose comprehension one must draw on abilities such as drawing inferences and extrapolating beyond the information provided in the text in order to decipher meaning. Both the reading and listening recall tasks are used by researchers today but some modifications have certainly been incumbent for these tasks to be suitable for use with younger children in terms of necessitating shorter sentences and relatively simple verification tasks (see Pickering & Gathercole, 2001).

One clear difficulty in the studies on the central executive has been the lack of ‘pure’ executive tasks. For instance, two of the most frequently used central executive tasks, random letter generation (e.g., Baddeley, 1966) and random number generation (e.g., Salway & Logie, 1995) not only disrupt the central executive but, due to the verbal production of the letters or digits, also impede the functioning of the phonological loop. Table 2 indicates a small selection of tests that purport to measure the central executive-complex working memory or executive function.

Table 2. Examples of tasks which measure CE-CWM and executive function

Author/s	**Task Type**	**Task Name**	**Designation**
(Stroop, 1935)	EF	Stroop Test	Inhibiting a prepotent response
(Wright, Waterman, Prescott, & Murdoch-Eaton, 2002)	EF	Animal Stroop	Inhibiting a prepotent response
	EF	Go/No Go	Inhibiting a response to a learned behaviour
(Swanson, 1999)	EF	Tower of London/Hanoi	Deficits in planning
(Jarrold & Citroën, 2013)	EF	Wisconsin Card Sorting Task	Set shifting
(Daneman & Carpenter, 1980)	CE	Reading Span	Storage and processing
(Alloway, Gathercole, & Pickering, 2004)	CE-CWM Verbal	Listening Recall	Verbal storage and processing
(Alloway, Gathercole, & Pickering, 2004)	CE-CWM Nonverbal	Odd One Out	Nonverbal storage and processing
(Alloway, Gathercole, & Pickering, 2004)	CE-CWM Numerical /Verbal	Backwards Digit Recall	Numerical storage and processing
(Alloway, Gathercole, & Pickering, 2004)	CE-CWM Spatial	Spatial Span	Spatial storage and processing
(Alloway, Gathercole, & Pickering, 2004)	CE-CWM Spatial	Mr X	Spatial storage and processing

1.5.3.3 A Note on the Fractionation and Development of CE-CWM

Fractionation of central executive complex memory spans in children has been discussed in relatively few papers (for examples see Alloway et al., 2006; Holmes, Adams, & Hamilton, 2008; St Clair-Thompson & Gathercole, 2006; Tsujimoto, Kuwajima, & Sawaguchi, 2007) and many of these papers refer to fractionation in the nonverbal mechanisms of short-term and working memory. St Clair-Thompson (2006) examined the two domains of complex memory span (verbal and visual) with a view to discover if they shared common or distinct links with other executive functions. That study reports that inhibition is dissociable from other executive functions in children, and that verbal and visual measures of CWM share a common association with updating, that is not linked to inhibition further supported by a paper in 2011 (St Clair-Thompson). The inferences drawn are that updating appears to be a domain-general process that is important to both verbal and nonverbal complex tasks and this also means that the dissociable aspects of verbal and nonverbal memory must come from other domain-specific components.

Some tests have been developed that allow us to discriminate between separable functions within CE-CWM. Whereas the listening recall task would be considered a measure of verbal working memory, a task such as the relatively new Odd One Out task (Alloway, Gathercole, & Pickering, 2004) would be deemed capable of measuring nonverbal (or visuospatial) working memory. There are now several tests that purport to measure nonverbal CE-CWM, such as Odd One Out, the Mister X/ Mr Blobby/Mr Peanut task (Alloway, Gathercole, & Pickering, 2004; de Ribaupierre & Bailleux, 1994; Hamilton, Coates, & Heffernan, 2003). For descriptive purposes the mechanisms of the Odd One Out task are considered. The task comprises two sets of three cards displayed concurrently. Two cards in each set are identical and the third card is the odd one out, is randomly assigned. On the third screen the card contents are removed but the card outline remains. The child then has to decide in the correct sequential order which two (increasing) card outlines contained the odd card out. The quantity of card sets increases by one set every six trials. In terms of storage and processing elements to this task comparisons can be drawn between this and both the listening and reading span tests. It fulfils the storage/processing criteria as the participant would have to make a processing judgement on which card was the odd one out, remember the spatial location of the first odd one out (storage), make an odd one out decision about the next set of cards (then any subsequent odd ones out) and finally recall the odd ones out in the correct sequence.

1.5.3.4 Development of CE-CWM

Developmental investigations of working memory have identified that complex span performance typically increases during childhood (Gathercole, 1999; Siegel, 1994; Towse, Hitch, & Hutton, 1998). Siegel (1994) discusses that the listening span task shows a constant steep developmental slope, continuing up to 16 years of age, a developmental profile that differs from short-term memory tasks which appear to show improvement up to the age of around eight years old, with a slight gradual increase occurring over the remaining years of childhood. This indicates that complex working memory may undergo a longer developmental period than the short-term counterparts and Gathercole (1999) discusses the possibility that the neuroanatomy of the brain and in particular the frontal lobe, might be an influencing factor in this rate of development. Further arguments put forth include a trade-off between processing and storage (Daneman &

Carpenter, 1980) and also that age-related changes were associated to the memory access and storage demands of the activities rather than processing demands (Swanson, 1999). Swanson argues on this basis that amount of activation of long-term structures changes with age, owing to increased availability of attentional resources as children grow older.

Given that there is such a close relationship between working memory capacity and other cognitive abilities in adults, it seems more than reasonable to presume that this growth in working memory performance might underpin much of the cognitive development experienced throughout childhood (Fry & Hale, 2000; Hitch, Towse, & Hutton, 2001; Leather & Henry, 1994).

1.6 Functional organisation of working memory in children

Several studies have attempted to further the understanding of the functional organisation of working memory (Alloway, Gathercole, Willis, & Adams, 2004; Gathercole, Pickering, Ambridge, & Wearing, 2004; Gathercole, Pickering, Knight, & Stegmann, 2004). It can be construed from Alloway et al (2004) that children aged between the ages of 4 and 6 have a modular working memory structure already in place and this includes verbal short-term memory and central executive – complex working memory. The authors also propose that the episodic buffer element is also in situ at this age range, but visuospatial abilities were not assessed in this paper. A second structural analysis of working memory also conducted in 2004 (by Gathercole, Pickering, Ambridge, et al.) did evaluate non-verbal STM, using the Block Recall Test and the Visual Patterns Test (Della Salla, Gray, Baddeley, & Wilson, 1997), yet the youngest group of children (4-5yr olds) did not complete listening recall, counting recall or Mazes Memory as the task demands were considered to be too difficult. Therefore, as the youngest cohort did not complete all the tasks they were ineligible for the analysis to identify a latent construct associated with complex working memory as each factor must be uniquely identified with at least two variables, but preferably with three as this will improve the chances of replication, and reliability. Given this information, while it is understood that there is a modular structure in place, and it does appear to fit well with the working memory model, it can't at this juncture assume that the same is true for central executive abilities in younger children. However, Gathercole et al (2004) did observe comparable results with the first structural analysis study, concluding that by the age of 6yrs old the three main components of the Baddeley and Hitch working memory model (1974a) are in position and each component increases in a linear fashion from the age of four up to early adolescence when it reaches maturity.

The following chapter will explore the empirical research which suggests that working memory and its theoretical components have a relationship with mathematics.

Chapter Two

Given what has been discussed about the theoretical background, the cognitive architecture, and the functional limitations of working memory in the previous chapter, understanding how these important memory systems are related to childhood mathematical attainment is clearly a very interesting research area. It may also be a key factor in helping us fathom the processes involved in attaining curricular mathematical competencies. In the long term also, this kind of research has the potential to be an influencing factor in curriculum design, teaching methods or mathematics strategies. This chapter will discuss how research into the field of working memory has been related to mathematics, with reference to ability and disability, scholastic attainment, mathematics development, and differing mathematical processes.

2 Working memory and mathematical research

2.1 Early investigations into the relationship between mathematics and working memory

Early working memory research focussed on the relationships between the phonological loop and language development and language comprehension however, more recently there has been an increase in research into the role of working memory in the mathematical arena. However, to put this into context it is necessary to go back a little further in time when initial papers regarding working memory and mathematics featured largely adult participants. Prior to Hitch's (1978) key text very little clear empirical evidence had been published, but plenty of speculative ideas had been put forward. Ideas such as, in written mathematics the visible page acts as a static workspace which would support, and even replace the mental working memory processes (Lindsay & Norman, 1972). Other authors suggested that mental arithmetic is limited by the requirement to hold information in the interim in a temporary working store (see Hunter, 1964; Kahneman, 1973; Lindsay & Norman, 1972). From Hitch's perspective it seemed both logical and pertinent to obtain some empirical evidence to support these mooted ideas. Hitch's work comprised a series of experiments, the first of which was somewhat exploratory. The participants were required to subjectively report how they completed the addition problems presented, with Hitch noting that all participants reported using a step-by-step strategy. This type of strategy is typically characterised by involving separate tens, units and hundreds additions to solve the problem posed. Only three of the 30 participants reported using an alternative strategy, but the alternatives were seemingly only employed when one of the addends facilitated easy "rounding up", then subtraction of the rounded up excess at the end of the calculation. As an example, 527 + 49 could be mentally converted to (527 +50 (-1)) in order to achieve the correct result. Hitch also noted many individual differences in the order in which the steps were carried out, and he speculated that this might have been indicative of rapid strategy selection based upon rapid estimation.

Hitch also made the claim that as these stages of calculation are occurring logic dictates that some kind of storage must be used in order to retain the early parts of the calculation, including carrying tens and units. He also stated that any loss of information in short-term mental storage would lead to errors in the calculations. Previous work hinted that errors in calculations may be caused by an overload of information in the limited capacity storage or working memory (Lindsay & Norman, 1972) and this was indeed evident in the Hitch study.

Following on from the idea that storage overload may impair performance, the dual-task methodology has been adopted frequently in adult studies (for a review, see DeStefano & LeFevre, 2004). This type of task requires the participants to perform two related tasks simultaneously in order to compare performance with single task conditions. It assumes that if a task selectively interferes with a particular type of processing but not with another type of processing, then those two types of processing must rely on different aspects of the cognitive system. These studies clearly show that working memory is needed in adults' simple arithmetic performance and more specifically, the performance of adults on simple arithmetic tasks seems to always rely on central executive/complex working memory resources, as opposed to verbal and visuospatial working memory resources (De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000). The evidence for the role of the phonological loop in addition and subtraction appears to depend upon the strategy used to complete the calculation as opposed to the operation under consideration. Studies that have looked at strategy use report that phonological load interferes with performance on tasks where counting strategies are used. (Hecht, 2002; Imbo & Vandierendonck, 2007b). Consistent with the idea that access to information stored in LTM, dual task studies have demonstrated that disruption of the phonological loop has not been shown to impair performance on single-digit multiplication problems, particularly for easy multiplication problems (De Rammelaere & Vandierendonck, 2001) .

The Hitch study was quite an exploratory piece of work, in theoretical terms that paper has been a precursor to large quantities of the subsequent research in adult mathematics and working memory and subsequent research with adults has expanded using dual task methods however, as important as these studies are, it is thought that some of the principles cannot be directly applied to studying mathematics and working memory in a childhood population. For instance, the inferences drawn from the Hitch (1978) study would require that the solution of a mathematical problem relies upon the participant having long term representations of mathematics facts and appropriate solution strategies. In one of Hitch's examples he cites the "estimation" strategy that adults adopt to facilitate the solution of a problem, but children may not routinely estimate until those long term representations are in place, either by taught methods or by experience.

2.2 Working memory in children with mathematical disabilities

As there is a specific concern about the educational development of children with mathematics disabilities research in this population has been a priority. Therefore much of the earlier work in the field was conducted with children exhibiting problems with mathematics (Geary, 1993; Geary, Bow-Thomas, & Yao, 1992; Geary, Hoard, & Hamson, 1999; Gross-Tsur, Manor, & Shalev, 1996; Swanson, 1993).

There are some common features that characterise a disability in mathematical development in terms of counting, number and arithmetic (see Geary & Hoard, 2001) and if we consider how a mathematical learning disability might impair mathematical performance there are a number of elements where problems may become apparent. It is possible that memory problems could interfere with the retrieval of basic mathematical facts. A child with a mathematical learning disability might also display marked impairments in abilities to solve word problems (Passolunghi & Mammarella, 2010), understand number systems (Geary, 2013), and use effective counting strategies (Gersten, Jordan, & Flojo, 2005). They may find it difficult to understand relationships between numbers, such as the ties between fractions and decimals, addition and subtraction, and multiplication and division. In terms of visuospatial issues, we could notice that children may misalign number columns, incur difficulty with place value in base 10, and find difficulty interpreting geometry, charts or maps and to further substantiate this Kyttälä and colleagues have recently shown that children with MD have a cognitive profile that shows deficits in visuospatial memory (Kyttälä, Aunio, & Hautamaki, 2010; Kyttälä & Lehto, 2008).

2.2.1 Incidence of MD in Children

It has been something of a challenge to obtain accurate data to indicate the official incidence of independent[2] mathematical learning difficulties in the UK. A 2004 parliamentary document estimates that the prevalence of dyscalculia lies between 1% and 7 % ("Postnote: Dyslexia and Dyscalculia," 2004). Butterworth reports that the incidence of the particular specific mathematical disability (dyscalculia) is between 5% and 7% in the UK (2002). Geary et al (Geary, Hoard, Byrd-Craven, & DeSoto, 2004) claimed that in the USA the figure for some form of learning disability in mathematics is between 5 and 8% of children. These statistics refer to children who are purely mathematically disabled with no comorbidity with other deficits such as dyslexia or specific language impairments. However comorbidity with spelling (Norway: 10.9% math disabled, 51% comorbidity with spelling disorder, Ostad, 1998), and reading disorders are common (Israel: 6.4%

[2] No comorbidity with any other language or cognitive impairment

dyscalculic, comorbity with reading disorder17%,   Gross-Tsur et al., 1996). There is some evidence that dyslexia also impacts upon mathematical development in children, but it has been suggested that the extent it impacts varies across different mathematical skills (Simmons & Singleton, 2008). It is suggested that dyslexic children exhibit difficulty answering arithmetic fact questions rapidly and correctly, but they perform at a similar level to peers on a test of understanding place value. Dyscalculics may also present with comorbid deficits in language, poor verbal skills, impaired working memory, and faulty visuospatial skills but these are not necessarily thought to be prerequisites to the condition. Although Shalev (2004) reports that mathematical disabilities do, in general appear as isolated and specific learning disabilities, such disabilities are also common factors in many other neurological or psychological disabilities. It is also not entirely clear how developmental dyscalculia interacts with other cognitive abilities, including memory, language, intelligence, and spatial and motor abilities. Arguably the clearest interaction is that of a double deficit of dyscalculia and dyslexia (e.g. Jordan, Kaplan, & Hanich, 2002), and if we consider mathematical word problems, and the language of math being so critical to comprehending such problems (Stock et al., 2009) then it is considerably more likely that people with this double deficit will experience more difficulties with process that involve both aspects of the deficit. Reports of prevalence of a general mathematics and reading disability vary from 17% (Gross-Tsur et al., 1996) to 43% (Badian, 1983) with the prevalence of combined mathematics and writing disabilities reported as about 50% (Ostad, 1998) however, given the earlier reported data that the prevalence of a significant problem with mathematical development may be between 2.5% and 4.3% of UK school age children (Geary et al., 2004), the figures of up to 50% seem somewhat inflated. Mathematical disabilities were also found to be comorbid with lower IQ, inattention as well as writing problems (Shalev, Manor, & Gross-Tsur, 2005), and symptoms of ADHD are often identified (Gross-Tsur et al., 1996) with over 20% of boys with ADHD also demonstrating mathematical disabilities (Faraone et al., 1993).

The literature regarding working memory and children with deficits in mathematics tends not to use the term dyscalculia and favours terms such as “mathematical disability” (Desoete, Roeyers, &

De Clercq, 2004; Geary, 2010; Swanson, Jerman, & Zheng, 2008) or “low arithmetical achievement” (D'Amico & Guarnera, 2005). There are a number of issues surrounding diagnosis of dyscalculia, and there is currently no formal diagnostic test for dyscalculia in the UK. Diagnosis tends to be based on the noticeable effects of dyscalculia leading to deficits in mathematical performance.

A further issue to consider is where would the cut-off point be in terms of a diagnosis? Butterworth’s Dyscalculia Screener (Butterworth, 2003) can be used as a diagnostic tool, but its use is limited to qualified practitioners, such as educational psychologists. Therefore the tendency appears to be to work with participants that consistently perform at the lower levels of mathematical attainment in schools, and refer to them as having a mathematical disability.

2.2.1.1 Use of immature strategies in arithmetic by mathematically disabled children

Considerable emphasis has been placed upon the strategies used by mathematically disabled children to solve simple arithmetic problems (Geary et al., 1992; Geary, Brown, & Samaranayake, 1991; Jimenez Gonzalez & Espinel, 2002). Examples of immature strategies might include finger counting or tapping, or remembering the answer by rote. These studies have indicated some consistent patterns in children with mathematics disabilities which are seemingly unrelated to intelligence.

It is a persistent key finding that children with mathematical disabilities will consistently utilise immature problem-solving procedures to solve simple arithmetic problems (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003), that are more typically associated with younger children without mathematical disabilities. As an example of this, the least mature approach to solving simple addition problems is called counting-all. So, in order to answer the question 3+2, younger children will hold up three fingers on one hand, counting "one, two, three, " followed by holding aloft two fingers on the other hand, counting "one, two,". To solve the problem they will then recount all of the aloft fingers starting from one. The more mature alternative is to articulate the largest number, three in this example, and then count-on a number of times equal to the value of the smaller number, as in “four, five” to achieve the correct answer. In the mature strategy the child would need to retain the larger figure in working memory whilst counting on to the result (Dowker, 1998; Hanich, Jordan, Kaplan, & Dick, 2001).

It is also apparent that remembering core arithmetical facts such as basic number bonds is characteristic problem for a child with a disability in mathematics, and this has two possible implications. Firstly, that there is a problem in the fact learning and retrieval process and secondly that there is a problem with inhibiting other facts. Geary and colleagues suggest that the problems with the fact learning/retrieval process, i.e. getting the arithmetic facts into long-term memory is akin to word finding difficulties in children with a reading disability (Geary et al., 1999). There is an argument that children who don’t have problems learning the arithmetical facts can still have problems with retrieving the correct fact as they can’t inhibit other numerical facts adequately. As an example, when attempting to answer a simple problem such as 4+5 a child with inhibitory deficit may recall 6 (next in the 4, 5, 6 counting sequence), they may also recall 20 (inability to recall the correct mathematical operation fact) along with the correct answer (Geary, 2010). Thus the time it takes to answer the sum is frequently longer, and the subsequent results are much more prone to temporal delay errors due to overloading working memory with too many facts being remembered. Theoretically the inference from this is that the central executive component of working memory may be related to this aspect of mathematical deficiency.

Geary (1993) showed associations between short-term memory and low attainment in both normally developing children and those with mathematical disabilities and also reported that children with mathematical disabilities had a digit span about one item lower than normally developing children (see also Geary et al., 2000). They concluded that poor memory span appeared to be a predictor of errors in mathematical computation as digit span negatively correlated with the frequency of computational errors. Furthermore, Geary et al (1999) hypothesise that children with mathematical disabilities are slower counters, thus resulting in calculation errors arising from information being lost though decaying traces. Geary additionally suggests that this causes the formation of weak associations between operands and their sums in long-term memory. Bull and Johnston (1997) extend this by suggesting that the speed of counting combined with weak associations in long-term memory are strong predictors of mathematical ability. Specifically, some of the deficits in mathematical ability can be attributed to the central executive as Bull and Scerif (2001) confirm , showing that maths ability correlates with executive function tests such as number stroop (inhibition), WCST (switching) and counting span.

In summary based upon much of the research with mathematics disabled children, it appears that working memory is strongly implicated in the problems that these children experience with mathematics.

2.3 Working memory and mathematics in typically developing children

Many psychologists have considered the underlying cognitive mechanisms that are involved in children's arithmetical development, and mathematics has been associated with all three components of the Baddeley and Hitch working memory model (1986b; 1974a) in typically developing populations. There are differing accounts of the levels of influence working memory has upon mathematics in children, as well as different components being shown to be involved in contributing to mathematics (examples of such research: Alloway & Alloway, 2010; Andersson, 2007; Bull & Espy, 2006; Bull et al., 2008; Bull, Johnston, & Roy, 1999; Gathercole, Brown, & Pickering, 2003; Holmes & Adams, 2006; Holmes et al., 2008; Iuculano, Moro, & Butterworth, 2011; Jarvis & Gathercole, 2003; Reuhkala, 2001; St Clair-Thompson & Gathercole, 2006). There are a number of reasons why I believe that differing accounts have been made, not least that age ranges of the participant children can vary greatly between studies, and therefore the children may be at different developmental stages of working memory. The working memory measures used also vary; some studies use a single working memory measure per WM component, whereas others use up to three tasks. In addition some studies only examine one aspect of the WM model. The country of origin of the study may also have an influence as curricular structure, and the onset of formal learning for children differs by country. Irrespective of these differences some common themes do occur and the remainder of this chapter will take each working memory component in turn and take notice of studies where the component in question has been implicated in mathematics in typically developing children.

2.3.1 Verbal Short-Term Memory (V-STM)

There is some evidence that verbal short-term memory skills are related to mathematics. De Smedt and colleagues (De Smedt, Janssen, et al., 2009) demonstrated that measures of the phonological loop were predictive of second grade mathematics, but not first grade and that VSTM mediates individual differences in single digit arithmetic, suggesting that the relationship may be an artefact of efficient arithmetic fact retrieval (De Smedt, Taylor, Archibald, & Ansari, 2010). Bull and Johnston (1997) reported that children with difficulties in mathematics tended to have problems specifically in automating basic arithmetic facts. They argued that this may arise from a general processing speed deficit rather than from a specific deficit in working memory as after controlling for reading ability arithmetic was best predicted by processing speed and this has latterly been corroborated in typically developing children by Passolunghi and Lanfranchi (2012). Earlier research had not typically included measures of processing speed and rarely accounted for reading ability. With this in mind Bull and Johnston (1997) proposed that other studies may have identified a relationship between general scholastic abilities (such as reading ability) and verbal short-term memory that is not particular to mathematics. However, in a French study Noël, Seron and Trovarelli (2004) made a link between working memory and calculation strategies in children aged 5-6. They showed that the working memory model, and in particular measures of phonological loop (VSTM) were significantly predictive of addition skills measured four months later. Also they noted that children with a higher capacity on phonological measures - were more accurate, and used more mature strategies and that processing speed did not predict addition skills. It was proposed that this is an artefact of better, faster retrieval from LTM and thus laying down stronger representations in LTM. They also argue that limited resources in phonological memory prevented children from using more sophisticated/mature strategies for solving addition problems.

Clearly there are some conflicting opinions and evidence about the influence of VSTM skill on mathematics and an issue to consider when thinking about verbal short-term memory and mathematics performance in children is the tasks utilised to measure VSTM. Many studies have used digit span as the primary verbal short-term memory task, and it has been discussed that children with an aptitude for mathematics may have stronger representations for numerical information (Dark & Benbow, 1990, 1991; Passolunghi & Siegel, 2001; Siegel & Ryan, 1989) and that as such they are prone to identifying and representing numerical information faster and more accurately, perhaps leading to an artificially high digit span (representing VSTM in the working memory model), or to overly strong relationships between mathematics and working memory (this is also discussed in Holmes & Adams, 2006).

2.3.2 Nonverbal Short-Term Memory (NV-STM)

Evidence suggests that nonverbal short-term memory is also associated with scholastic attainment, supporting mathematical processing in children (Gathercole & Pickering, 2000b; Jarvis & Gathercole, 2003; Mayberry & Do, 2003; Reuhkala, 2001). In a study that was related to the UK National Curriculum, Gathercole and Pickering (2000b) reported that children with low levels of early curricular attainment, including mathematics, also displayed deficits in NVSTM and CE-CWM. Reuhkala (2001) identified visuospatial skills as being influential on scoring on a Finnish National Mathematics test, but verbal central executive capacity and phonological working memory were not significantly related to maths in 9th graders (age 15-16). Reuhkala considers that mathematical operations may be supported by visual imagery and as such by visuospatial memory. This is substantiated to some degree by a number of other researchers who also advocate that visuospatial memory may provide a mental workspace for maths when children are using mental imagery as a back-up strategy for performing mathematical tasks (Bull et al., 1999; Holmes & Adams, 2006; Holmes et al., 2008). In another study with older children (aged 15-16) visuospatial short-term memory displays a relationship with overall mathematics performance, and that different parts of visuospatial memory were associated with different types of mathematics (Kyttälä & Lehto, 2008).

Holmes and Adams (2006) research is the closest precursor, and possibly the most relevant to this thesis, as the authors researched working memory, typically developing children, and the UK mathematics curriculum. Their primary analyses identified that performance on measures of visuospatial sketchpad and central executive, but not phonological loop, predicted unique variance in children's curriculum-based mathematical attainment, but the relative contributions of each component did not vary much across the different mathematical domains. Their research also indicated that the WM processes supporting children’s mathematics change with age, and consequently define possible independent roles for the two slave systems. These different roles seem to demonstrate a shift from early visuospatial strategies to mature, verbal solution strategies such as direct retrieval. This configuration of results is consistent with McKenzie et al. (2003) who established that younger children use visuospatial strategies in mental arithmetic, while older children use a combination of verbal STM and nonverbal STM strategies. Furthermore, recent evidence from Nyroos and Wiklund-Hörnqvist (2011) investigating associations between WM and educational attainment in different mathematics subtopics in Sweden has found evidence that in overall mathematics children appear to depend upon visuospatial codes but this association tends to decrease in favour of a reliance on phonological ability to combine both verbal and visual codes.

2.3.3 Central Executive - Complex Working Memory (CE-CWM)

Pertaining to the UK, Bull, Espy and Wiebe (2008) published a study into short-term and working memory as longitudinal predictors of mathematics. They measured pre-schoolers on a battery of executive function, complex working memory and short-term memory tests including backwards and forwards Digit Recall, backwards and forwards corsi blocks, as well as Tower of London, and Shape School (measuring executive function). Mathematics and reading was measured using Performance Indicators in Primary Schools (PIPS) which is an information system that tracks a number of aspects of schooling as the pupils move through the primary sector, comprising of a series of academic attainment quizzes, measuring basic and more complex mathematical skills. In this study children were tested three times in a three year period, with the cognitive tests undertaken only at the first testing phase. Bull et al found that at phases 1 and 2 (ages 4.99 yrs. and 5.70 yrs.) all the memory and executive function tests correlated with the maths outcome, but at the third and last time of testing (aged 7.71 yrs.) both short-term memory measures failed to show any association with maths. Regression analyses added to this by showing that at the final testing stage the only significant independent predictor of mathematics was a central executive working memory span task. Bull et al believe that this demonstrates the importance of working memory in mathematics achievement and that the capacity to process and store material in working memory significantly impacts on a child's ability to acquire skills during the early period of formal education.

Gathercole and colleagues have conducted a number of investigations into the relationships between working memory and curricular mathematics with children ranging from age seven to age fourteen, obtaining a broadly similar pattern of results (Gathercole & Pickering, 2000b; Gathercole, Pickering, Knight, et al., 2004; Jarvis & Gathercole, 2003). Gathercole and Pickering (2000a) suggested that central executive scores uniquely predicted mathematics scores at 8 years old. In a closely related study, Gathercole and Pickering (2000b) reported an association between working memory abilities and National Curriculum attainment at age 7. Participants were allocated to normal and low achieving groups based upon their Key Stage 1 National Curriculum test scores in Mathematics and English, and working memory was assessed using an early version of the WMTB-C (Pickering & Gathercole, 2001). Children who were not reaching levels of normal curricular attainment showed distinct impairments on both NVSTM and CE-CWM. These results indicate that working memory, and in particular the visuospatial sketchpad and central executive, support curricular progress at age 7, particularly demonstrated in mathematics (Gathercole & Pickering, 2000a). A similar study by Gathercole and colleagues (Gathercole, Pickering, Knight, et al., 2004) also explored the relationship between National Curriculum assessments in mathematics and working memory, and found that in children aged 7 and 8 years old central executive scores were again significantly associated with mathematics. In each study, at each age group examined, working memory is found to be significantly associated with curriculum assessments of mathematics. Moreover it was found that tasks tapping into complex working memory span functioning (central executive) were more significantly associated than tasks measuring verbal and nonverbal short-term memory. It should be noted that Gathercole and colleagues utilised curriculum assessments that were developed by the QCA and scoring on these tests is banded, therefore pupils were categorised into levels and there is a possibility that this mathematics measure may not have been as sensitive as the raw scores. It is also noted that this measure becomes a global score of mathematics and thus disentangling the effect of working memory upon mathematics is quite limited. Of strong interest is the study conducted with children aged 11 and 14 by Jarvis and Gathercole (2003) which not only established a significant association between working memory and mathematics. However they discuss the finding that nonverbal complex working memory (Spatial Span) was most specifically related to curricular mathematics, playing a vital role in the acquisition of complex cognitive skills.

In summary, converging evidence indicates an association between working memory ability and performance on National Curriculum tests. In order to properly assess mathematical skills attained by children that participated in this project, this thesis has adopted a uniform standardised mathematics test that is part of a connected series of ten tests. It was chosen as it closely maps on to the UK National Curriculum and National Numeracy Strategy, and latterly the Primary National Strategy, and it measures the skills that children in the UK should be being taught at each age range throughout the duration if the project. To this end this study defines mathematical ability by adopting the four key concepts that are taught to children in UK primary schools and classified by the NNS as "strands". The four key concepts are Number, Calculation, Problems Solving, and Measures, Shape, and Space and each of these concepts will be discussed in more detail in Chapter 3.

2.3.4 Broad Aims and Hypotheses

1. There is a testable hypothesis in that the preceding research predicts a relationship between curricular mathematical skills and working memory (Holmes & Adams, 2006).

2. It is also anticipated that there will be a specific relationship between the central executive and curricular mathematical competencies (Bull et al., 1999; Bull & Scerif, 2001; De Smedt, Janssen, et al., 2009; Holmes & Adams, 2006; Jarvis & Gathercole, 2003).
3. There is a large element of exploratory work within this research. There is a possibility that the research may be able to indicate which specific areas of maths within the National Curriculum guidelines are particularly affected by problems with working memory. Based on previous research the key predictions are that central executive will be a key factor in performance on both the Calculation (similar to Berg, 2008) and Problem Solving strands.
4. This thesis also wanted to take account of the longitudinal predictors of UK curricular mathematical attainment. It is postulated that the central executive component of working memory would be most predictive of UK curricular mathematics over time (Bull et al., 2008; Bull & Scerif, 2001; Holmes & Adams, 2006).

Chapter Three

This section outlines the key points of the National Curriculum, National Numeracy Strategy and the subsequent Primary National Strategy. The relevance of this section will become increasingly apparent in later chapters where the relationships between the curriculum and strategies and working memory are further teased apart.

3 National Numeracy Strategy and Primary National Strategy

The implementation of the National Numeracy Strategy (NNS) occurred in September 1999 and was followed by several years of inspections, research and test evidence from schools in England. It was devised as a follow on to the National Curriculum (DfEE & QCA, 1999b) with the objective of 75% of eleven year olds reaching a national standard of mathematical attainment by 2002. The Strategy ensured that all schools routinely provided a daily structured mathematics lesson lasting forty-five minutes to one hour for all primary age pupils. Within this daily session the teacher spends the majority of the time “whole class” teaching, and a strong focus is placed upon mental mathematics.

3.1 Framework of the NNS and the “Strands”

A key document for teachers in the provision of the daily structured mathematics lesson was The National Numeracy Strategy: Framework for teaching mathematics from Reception to Year 6 (DfEE, 1999). This prescriptive document contains yearly teaching programs for each age grouping including guidance on the daily mathematics lesson, objectives planning grids and examples for the teachers to follow. The mathematics programmes of study as detailed in The National Curriculum (DfEE & QCA, 1999b) and the National Numeracy Strategy Framework for teaching mathematics are fully aligned. Whereas the National Curriculum (NC) document sets out the legal requirements of the National Curriculum in England for mathematics, the Framework provides a detailed basis for the implementation of the statutory requirements of the programme of study for key stage 1 in mathematics. The framework comprised of five key strands:

- Numbers and the number system
- Calculations
- Solving problems
- Measures, Shape, and Space
- Handling Data

During Key Stage 1 the first three strands have direct links to the NC programme of study for "Number" (page 16), the fourth is linked to "Measures, Shape, and Space" (page 19), while Handling Data is not directly linked to any Key Stage 1 programme of study, but is gradually introduced to help provide a foundation for handling data at later Key stages. The strands are described separately in the NNS, yet the connections between each strand span many topics throughout the teaching. The Framework stresses that in their lessons and lesson plans teachers should provide examples and activities explain and demonstrate these connections to children, thus fully integrating mathematics throughout the curriculum.

3.1.1 Numbers and the number system

This strand considers the basics of understanding number, counting, and the properties of number and number sequences, including negative numbers. It also covers place value and ordering, ensuring that pupils can both read and write numbers. Moreover, estimating and rounding also have focus, as do fractions, decimals and percentages and their equivalence ratio and proportion. Learning outcomes for this strand are displayed in Appendix A.

3.1.2 Calculation

The Calculation strand covers understanding number operations and relationships and rapid mental recall of number facts. It also deals with mental calculation, including strategies for deriving new facts from known facts. While there is a strong focus on mental strategies the strand also covers pencil and paper methods and the use of calculators, and encourages pupils to incorporate checking that results of calculations are reasonable. The learning outcomes for this strand can be found in Appendix B.

3.1.3 Problem Solving

The topics that are covered by this strand include making decisions about which method of calculation is appropriate to use; reasoning about numbers and making general statements about them. It also introduces solving problems in "real-life" contexts. Details about the learning outcomes for this strand are in Appendix C.

3.1.4 Measures, Shape and Space

The themes managed by this strand include choosing units and reading scales, such as rulers and choice between inches or centimetres, the properties of two and three dimensional shapes, position, direction and movement (see Appendix D for learning outcomes).

3.1.5 Handling Data

The Framework suggests that this strand should be about collecting, presenting and interpreting numerical data; however this aspect is not fully applied until Key Stage 2 and is not discussed at any length within this thesis.

To further support the teaching and learning of primary mathematics a series of key objective and core learning targets are clearly defined in the NNS for each academic year grouping.

3.2 National Numeracy Strategy Evaluations

Since the implementation of the NNS in 1999 there have been several evaluations conducted to examine the efficacy of the Strategy. Two reports from UK sources (Minnis & Higgs, 2001; Ofsted, 2002) strongly suggest that National Numeracy Strategy as a whole has achieved an increase in standards and provided benefits to teaching mathematics to children. Minnis and Higgs (2001), commissioned by the Qualifications and Curriculum Authority focussed heavily on testing the children and obtaining performance ratings based upon age-standardised scores. Their data shows that, in 1999, prior to the introduction of the NNS, 56% of children tested in Year 4 achieved a national curriculum Level of 3 or better but in 2001 the proportion of children at this level had increased to 70%. At the same time there was a reduction in the proportion of pupils failing to reach the lowest level measured by the evaluation tests. For Year 3 pupils there was a 5%

reduction (from 15% to 10%) in those working below Level 2. Clearly this kind of evidence demonstrating the vast improvements shows that the targets that were set out in the NNS are achievable. However in the Minnis and Higgs report it is interesting to note that in 2001, pupils who had achieved lower levels at Key Stage 1 had made relatively more progress than higher achieving pupils by the time they took the evaluation tests. This suggests that the gap between lower and higher achievers was narrowing during Key Stage 2, possibly as a result of the NNS teaching strategies. Minnis and Higgs accept that they are unable to assess the impact of the NNS but conclude that overall standards have improved during the first three years of implementation.

The Ofsted report (2002) delves into more practical aspects of the implementation of the NNS. One of the key findings of the report is that the oral and mental aspect of the lessons remains the best-taught element of the daily mathematics lesson. Aided by good teaching standards number fact recall has improved, yet the report claims that teachers are not able to give sufficient attention to the teaching of mental calculation strategies. Another concern and point for action is that there are weaknesses in the teaching during the plenary session and a noticeable effect of this is that teachers focus on one groups problem area, leaving the children who are able in that topic, bored and uninterested. This concurs with the findings of Minnis and Higgs (2001) providing practical reasons as to why the gap between poorer pupils and more able student is lessening.

Prior to August 2002, children entering Reception class were assessed using baseline assessment testing, however the QCA has recently made advances in how children of this age group are assessed. No longer do children undertake baseline testing, instead the child is concurrently assessed during the academic year. This causes some difficulty for this research, as no longer is there a baseline assessment score to indicate the participants' academic level at the commencement of the research. Furthermore, access to SAT data at the end of the project was not allowed by the schools.

3.3 The Introduction of the Primary Strategy

In May 2003 (DfES, 2003a) the Primary Strategy was introduced to supersede the National Numeracy Strategy. For the most part the introduction of this document was to maximise the successes of the Numeracy Strategy, to aim to make improvements to the weaker aspects and to provide a more cohesive curriculum. An aspect that many teachers have since reported in personal communications as being beneficial has been that the Primary Strategy allows some more freedom and flexibility for taking account of individual difference in children's learning styles, and also allows for more creative teaching methods to be employed.

A clearer framework for teaching mathematics has been provided by simplifying the structure of the objectives for teachers. The Primary Strategy now identifies seven strands of learning to give a broad overview of the mathematics curriculum in the primary years. Objectives are associated with the seven strands to demonstrate progression in each strand. The seven strands are not equally weighted. In constructing the strands, knowledge of number facts has now been separated from calculation, methods of calculation have been combined, measures have been kept apart from shape and space, and problem solving has been embedded into the broader strand of using and applying mathematics. The seven strands relate very readily to the 1999 Framework and the programmes of study in the National Curriculum for mathematics. Covering the objectives in the seven strands will support children in their progression towards the Early Learning Goals. According to the DfES (2003a) the construction of the newer Framework around the seven strands not only simplifies the overall structure, but also presents a more useful tool for highlighting and amending some of the aspects of mathematics that children find difficult to learn.

For the purposes of this thesis reference to the strands will be to the earlier inception with four of the five key strands identified in the National Numeracy Strategy documentation (Number, Calculation, Measures, Shape and Space, and Problem Solving). The rationale for this is because the mathematical assessment used in the study was broken down into those four strands, and they can be quite readily mapped on to the strands identified in the subsequent Primary Strategy.

Chapter Four

The methodology for this thesis was the same at each year of testing. Therefore a decision was made to describe the design, participants and procedure in one chapter for easy reference.

4 Methodology

4.1 Rationale

This research was designed to complement and extend previous research that has focussed specific measurable mathematical skills (e. g. addition (Kalaman & Lefevre, 2007; Noël, 2009; Noël, Seron, & Trovarelli, 2004), multiplication (Robert & Campbell, 2008; Seitz & Schumann-Hengsteler, 2000) arithmetic problem solving (Andersson, 2007; Fuchs et al., 2006; Swanson, 2011; Zheng, Swanson, & Marcoulides, 2011)) by using a mathematics test that maps directly on to the curriculum as it is taught in UK schools. This test enables the cognitive skills that are relevant to mathematics as taught in UK classrooms to be identified. There is very limited previous work that has examined the influence of working memory involved in curriculum areas such as Number or Measures, Shape and Space. The study design extends prior work that has studied the relationship between different areas of curricular mathematics and working memory, principally Holmes & Adams (2006) and Holmes et al. (2008). It extends this work in a number of ways including the adoption a more complete assessment of working memory. This was achieved by including two measures each of verbal short-term memory, nonverbal short-term memory and central executive-complex working memory span. Furthermore the study design allowed the data to be analysed longitudinally.

For completeness of design, quantifying the cognitive predictors of mathematics involved longitudinal analyses. Bowey (2005) suggests that small group studies, single class studies and fewer than 50 children may be unreliable. Therefore this study recruited participants from two schools, each with two classes, and it tests in excess of 50 children. Therefore the design is considered to be adequately robust. Several studies have previously considered the longitudinal cognitive predictors of mathematics, but frequently they have looked at children with a mathematics disability or a propensity for low mathematical attainment (Bull et al., 2008; De Smedt, Janssen, et al., 2009; Geary, 2011; Stock, Desoete, & Roeyers, 2010; Swanson, 2011; Vukovic, 2012). There is very little literature to suggest that any have examined UK curricular mathematics specifically apart from Holmes and Adams (2006) and in general the duration between testing times has often been quite short, although this is not always the case (Vukovic, 2012). The design of this study meant that the children were studied over a crucial three year period of their early working memory development and mathematics education.

4.2 Design

This thesis utilized a descriptive multivariate correlational design to determine the relationships between working memory components and mathematical ability annually, over a three year period. Data was further analysed using forced entry hierarchical regression models.

4.3 Participants

The participants were all typically developing children recruited from two primary schools in the North-West of England and were tested on the same measures annually for three years. Schools were invited to participate by letter, and when interest was expressed a follow up visit made to the school. This ensured that the school was willing to commit to a three year project and understood the scope of the research.

Table 3. Mean age in months (to 2dp) at time of testing (N=70)

	Reception		**Year 1**		**Year 2**	
	$\bar{x}$	**(SD)**	$\bar{x}$	**(SD)**	$\bar{x}$	**(SD)**
Age in Months	61.00	(3.75)	72.74	(3.93)	83.53	(3.58)

In the first year of testing 87 children in from the Reception classes participated, 76 in the second year (the children were now in Year One), and this reduced to 70 children in the third year (Year Two), however only the complete data from the final cohort of 70 children were used in the analysis. Incomplete tests were provided for 17 children overall. Reasons for incomplete data were refusal or inability to complete the test (n=3), absence through illness or holiday (n=11) and moving from the school catchment area (n=3). The gender split in the final cohort of 70 participants was 34 males and 36 females (This data is represented in Table3).

Consent was sought from parents, and children with confirmed statements of special educational needs were excluded from the study. All children began full time schooling in September 2003. All measures were completed during January to June each year for three years with School One comprising the January to March period, and School Two taking from April to June. Only data from children who had completed all parts of the test battery were included in the final analysis.

4.4 Equipment

A portable Compaq Evo Notebook was equipped with software required for the presentation of images and recording responses. An LCD screen (800x600 screen resolution) was positioned approximately .75m away from the child's face on a desk. Installed on the notebook computer was the Automated Working Memory Test Battery for Children V1.0 (Alloway, Gathercole, & Pickering, 2004). This battery is based heavily upon the Working Memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001), with several well-known measures of working memory, and the introduction of one novel measure (Odd One Out).

4.5 Procedure

The children participated in three testing sessions at each year of testing and each child was tested individually in a quiet area of the school. In the first session two verbal short-term memory tests (V-STM), one visuospatial short-term memory test (NV-STM) and two working memory tests (CE-CWM) were administered from the Automated Working Memory Assessment (Alloway, Gathercole, & Pickering, 2004) (AWMA). The AWMA tasks were completed in a fixed sequence in order to vary task demands across successive tests. Given that some studies (Lehto, 1995; Passolunghi & Siegel, 2001) have reported a strong association between span measures with a numerical basis (e.g. digit span, counting span, backwards digit span) and mathematics, Digit Span from the AWMA was also assessed. This measure was taken to allow for the statistical examination of this reported association at each year of testing. However, in line with Holmes and Adams (2006) this study did not make use of numerical span tasks in order to assess any link between short-term and working memory and mathematics independent of any direct access to number, or number knowledge.

In the second session, pairs of children were administered age appropriate mathematics assessments from the Mathematics 5-14 series (NFER-Nelson, 2001). During the third session at the Reception time of testing each child was individually tested on the Mazes Memory from the Working Memory Test Battery for Children (Pickering & Gathercole, 2001). In the two years subsequent, Object Assembly and Block Design subtests from WISC-R (Wechsler, 1974) were taken as measures of non-verbal intelligence, these are subsequently collectively referred to in this text as "Performance Measures".

4.6 Working Memory Measures

4.6.1 Working Memory Scoring System and Convergent Validity

The AWMA calculates the correct number of responses according to a key press initiated by the experimenter following the participant's response. The scores given by the AWMA are the number of trials correct and the standardised score. However on the beta version of the AWMA this function was not available so the scoring was calculated by hand. As the participant responded correctly a score of 1 was awarded, an incorrect response elicited a score of zero. Upon four correct responses the software automatically moves on to the next block of trials giving credit for the two omitted trials (thus a maximum score of 6 correct per block). The computer task terminates upon three incorrect responses in any single block. Raw scores were computed by summing the number of items correct, plus any credit given for the omitted trials from completed blocks.

Convergent validity for the AWMA is reported in Alloway et al (2008) where the AWMA is described as having a high degree of convergence in performance with the WISC-IV Working Memory Index .

4.6.2 Central Executive - Complex Working Memory (CE-CWM)

Two complex working memory tasks were administered from the AWMA (Alloway, Gathercole, & Pickering, 2004) to tap into different areas of working memory functioning, verbal working memory and nonverbal working memory.

4.6.2.1 Listening Recall –Verbal CWM

The Listening Recall task verbally presents short sentences with a spoken duration of approximately 1 to 2 seconds. Some sentences are true statements whilst others are untrue. Immediately after the verbal presentation the participant is prompted to judge the accuracy of the statement with a true or false response, and then recall the final word from each of the sentences, spoken in the exact order heard. Credit is only given if the participant fulfils both the validation and recall accurately. The first block of six trials begins with a single sentence, progressing by an additional sentence per block of six trials. The test is terminated upon three incorrect responses in a block and moved on to the next block after four correct trials (for more detailed scoring information see below). Test–retest reliability is .81 (Alloway et al., 2008)

4.6.2.2 Odd One Out – Nonverbal CWM

The Odd One Out task was administered as a measure of non-verbal central executive. The software visually presents a set of three rectangles each containing a simple shape. One out of the three shapes is odd. The participant must indicate which shape out of the three is the odd one out, then all of the shapes are removed leaving behind blank rectangles. The child must then recall which rectangle contained the 'odd' shape. The first block contains 6 trials of a single set as described; this is then increased by one set every 6 trials, therefore the second level would show two sets of odd ones out before progressing to the set of blank rectangles. At this point the child must identify where the first odd one out shape was, then where the second odd one out shape was in the correct sequence. The move on and discontinue rules are identical to those above. Test-retest reliability is .81 (Alloway et al., 2008)

4.6.3 Verbal Short-term Memory (V-STM)

Three tests of verbal STM were administered from the AWMA (Alloway, Gathercole, & Pickering, 2004). However Digit Recall was only measured in order to be able to examine for a specific strong association between short-term and working memory and mathematics. Therefore Digit Recall is not included in the regression analyses.

4.6.3.1 Word Recall

Word Recall verbally presents words for immediate serial recall. The words are presented at a rate of one per second and participants are required to recall the list of words in correct serial order. The list length begins with a single word and increases by one word every six trials. The move on and discontinue rules apply for all of the remaining AWMA measures; this rule is as described in 4.6.1. The maximum raw score of items correct is 36. Test-retest reliability for the Word Recall task is .76 (Alloway et al., 2008).

4.6.3.2 Nonword Recall

Nonword Recall follows the same procedure as the Word Recall task with the use nonsense words in place of real words. The sequence of nonwords must be recalled in the correct serial order. Test-retest reliability is .64 (Alloway et al., 2008).

4.6.3.3 Digit Recall

Digit Recall also follows the same procedure as the Word Recall task with the use numbers in place of words. The sequence of digits must be recalled in the correct serial order. Test-retest reliability is .84 (Alloway et al., 2008).

4.6.4 Non Verbal Short-term Memory (NV-STM)

4.6.4.1 Mazes Memory

Two nonverbal STM measures were administered. The Mazes Memory task was administered from the WMTB-C (Pickering & Gathercole, 2001). The Mazes Memory task can be presented in a dynamic or a static fashion and this study opted to utilise the static method to ensure a balance between the two NV-STM tasks. In the static version (Pickering et al., 2001) the administrator shows the participant a route indicated in red on a two-dimensional maze for duration of approximately 3 seconds. The child is directed to recall the exact route traced on a response sheet showing an identical blank maze. The first maze has a stick man in the centre and two rectangles around the man, each with a gap to allow a route out of the maze. At each span level, the complexity of the maze to be remembered is increased by one extra rectangle. Completing four items at each span level allows progression to the next span set and failing three items at one span level terminates the subtest. Test- retest reliability for Year 1 and 2 children is .68.

4.6.4.2 Block Recall

Block Recall presents a video clip of a finger pointing at a block(s) on a typical block design board, the video clip ends but the blocks remain in position. The participant is asked to recall the correct serial order of the indicated blocks by pointing at the position on the screen. As with the previous tasks, the sequence begins with a single block to recall and after each 6th trial the sequence increases by one block. The test is automatically discontinued following three incorrect responses in any block. Scoring is as above and test-retest reliability is .64. This task is considered to be a dynamic/spatial task as each block is [dynamically] tapped one at a time and can only be identified on the basis of its spatial location (Pickering et al., 2001).

4.7 Mathematical Measures

4.7.1.1 Mathematics 5-15 Test Series

Mathematics ability was assessed using the Mathematics 5, 6 and 7 assessments from NFER-Nelson (2001), administered at the age appropriate year grouping. The tests are verbally administered so that limitations in the reading ability of pupils do not mask the assessment of their mathematical attainment. The questions address Level 1 of the National Curriculum and close reference is made to the National Numeracy Strategy (DfEE, 1999). The series is also highly correlated with the end of Key Stage mathematics tests and thus suited to estimating the end of Key Stage results. These tests are suitable for administration at any time during the school year and were administered to pairs of children; having a duration of between 30 to 50 minutes. Each test is administered in a prescribed format and scored with 1 for a correct answer and 0 for an incorrect answer.

4.8 Performance Measures

4.8.1.1 Non-verbal Ability Subtest Measures

As in Gathercole et al (Alloway et al., 2005; Alloway, Gathercole, Willis, et al., 2004; Gathercole, Alloway, Willis, & Adams, 2006) two subscales (Object Assembly and Block Design) from the Wechsler Intelligence Scale for Children – Revised (Wechsler, 1974, 1977) were administered to provide a Performance Measure score for each child. Test-retest reliability coefficients for the ages 4-6 years range between .56 and .70 for the Block Design test, and between .84 and .87 for Object Assembly.

4.8.1.2 Block Design

This sub-test uses a set of cubes that are coloured red on two sides, white on two sides and red and white on the remaining two sides, split across the diagonal of the face. The experimenter has a booklet with eleven designs that should be completed in order by the participant. When administering this test to the youngest age group (age 6-7) the first two designs are demonstrated using the cubes, subsequent designs are reproduced by the participant from the booklet. The test should be discontinued after 2 consecutive failures. A failure would be where the time limit is exceeded, where the design is more than 30 degrees in rotation from the design pattern, or an incorrect representation of the pattern. Time limits vary according to the complexity of the design ranging from 45 seconds to 120 seconds. Scoring includes time bonuses for speedy completion.

No credit is given for partially correct or incomplete performance. The maximum score is 62 points.

4.8.1.3 Object Assembly

The Object Assembly sub-test contains five boxes each with a simple jigsaw style cut out. Administration begins with a sample item of an apple, followed by a child (girl), a horse, a car and a face (male). There are an increasing number of joins in each subsequent cut out and each must be set out, behind a shield in a specific layout. Again timing increases with complexity of the puzzle and time bonuses are awarded for faster completion. Scoring for item 1 is equal to the number of cuts correctly joined plus a maximum bonus of 2 points for perfect, fast performance. For the horse it is equal to the number of cuts correctly joined plus a bonus of 3 points for perfect performance. The scoring method for the final two items is one half the number of cuts correctly joined, plus a maximum of 3 time bonus points. A cut is considered correctly joined even if the segment made is not joined to the rest of the object and credit should be given for that join. The maximum score that can be awarded is 33 points and time bonuses are only awarded for perfect assemblies.

On both performance ability tasks raw scores can be converted to scaled scores appropriate for the age of the child as defined in the tables of the WISC-R manual. However, the sub-test score should typically be based on the use of four sub-tests. Given that time restrictions and financial constraints prevented the administration of the full performance ability sub-tests raw scores for each test administered were used.

Chapter Five

In this chapter the relationship between working memory and overall mathematics is examined over a three year period. In addition, the specific reported strong associations between a numerically based working memory task and mathematics are statistically analysed and reported.

5 Working Memory and Mathematics

As already discussed, research into the cognitive processes that underpin mathematical ability have been widely reported in recent years, however there is still a lack of cohesive research in this area with very young school children and pre-schoolers, and especially when pertaining to any school curricula. Several issues have hindered progress in examining children of the age range investigated in this thesis. The kind of maths assessments available that are suitable for the developing population are wide ranging, tapping into many different aspects of mathematics, and very few can measure the ability of the child on a contiguous scale. Moreover, the difficulty of the majority of the complex working memory tasks has also delayed research with very young children. As working memory capacity is so small at the youngest age range in this study, introducing an additional processing factor to a working memory task often renders the task unfeasible. At the commencement of the present longitudinal project a lack of normative data for the newer, more appropriate working memory tests was also problematic.

A further issue confounding the full understanding of how mathematics and working memory are related is that a significant proportion of the research that is available for this age range assesses children who already have a propensity for cognitive dysfunction, like children whose birth was premature (Hack, Klein, & Taylor, 1995; Hunt, Cooper, & Tooley, 1988); or those children with language and comprehension problems and attentional or learning disabilities (Siegel & Ryan, 1989; Swanson, Ashbaker, & Lee, 1996). From the broad research completed with children who have MD problems it can be inferred that children with MD often exhibit significant deficits in some WM functions. Several researchers have looked at dyscalculic children and children exhibiting deficits in mathematics, and a pattern emerges of children with shortfalls in visual and spatial skills (Rourke, 1993; Temple, 1991; Temple & Sherwood, 2002). Geary identified this visual and spatial deficit as a subtype of mathematical disability in 1993 indicating that there are possibly more subtypes of mathematical disability. Typically these papers highlight that those children with such a visual and spatial deficit will likely reveal such problems as misalignment of columns, issues with mathematical symbols (+ and -), as well as the general organisation of numbers and number patterns. Other studies have shown that mathematically disabled children do not retrieve as many facts directly from long-term memory (Geary, Widaman, Little, & Cormier, 1987; Ostad, 1998). Therefore complex working memory may play a key role in their solution of mathematical problems when direct LTM retrieval is not forthcoming as the mature solution strategy (Bull & Johnston, 1997; Geary & Brown, 1991; Geary et al., 1991) indicative that these deficits in working memory processes can be suggestive of a problem with mathematical functioning in children.

5.1 Typically Developing Children

In terms of examining the cognitive aspects of working memory in typically developing children Bull, Johnston and Roy (1999) showed that children with low mathematical abilities, but not a specific diagnosed mathematics disability, tended towards having significant deficits in some working memory functions. Their main finding was that the low ability mathematicians in the study performed significantly worse on Wisconsin Card Sorting Test, finding it hardest to sustain attention on this recognised measure of executive functioning. However Bull and colleagues found no significant differences between high and low ability maths with regard to measures of visuospatial working memory. As this cited research was conducted with children aged around 7 years old, the results while highly relevant and interesting cannot necessarily be extrapolated directly to children of a younger age range. However there is a degree of correspondence with the age of the children in the Bull et al study and those in the present study, as the participants in the current study reached circa age seven in the latter portion of the testing period. Any inferences should be drawn with the caveat that children of school age are still developing their WM skills over a long period of time, and while we understand a broadly linear developmental trajectory, it may be the case that hiccups can occur in that trajectory. Some recent studies with children similarly aged to the children in the present thesis (Nyroos & Wiklund-Hörnqvist, 2011; Simmons, Willis, & Adams, 2012) discuss the notion that different aspects of working memory have different patterns of association with mathematical skills. Simmons et al found that nonverbal STM predicted unique variance in magnitude judgements and number writing, but CE-CWM span tasks were better suited to explaining the unique variance in performance on additions accuracy in children, potentially demonstrating an important role for complex working memory spans tasks in relation to calculation type tasks. Nyroos and Wiklund-Hörnqvist (2011) took account of six different mathematical domains, written arithmetic, mental arithmetic, time, number understanding, fractions, and area and volume. Of each of those domains working memory was a significant predictor, apart from written arithmetic. Typically we would expect working memory to be somewhat associated with written arithmetic as it is a task that usually requires a calculation to be made, so this result is a little unusual. Nyroos and Wiklund-Hörnqvist regard the lack of WM support for this domain as being due to rote or imitative learning of algorithms and that would suggest that the pupil is not learning what to do in these type of task, but how to do it and as such there is not necessarily any conceptual understanding of the task. However, it could be argued that written mathematics requires a smaller contribution from working memory than mental arithmetic does, due to the on-going visual support of the numbers and operations on the paper as opposed to the mental storage of those while performing mental arithmetic.

Another study working with typically developing older children (aged 10 to 12yrs) by Imbo and Vandierendonck (2007a) hypothesised that mathematics would utilise more working memory resources in the earlier stages of learning, until more robust long-term representations such as number bonds and pairs, and number facts are made. They also make a claim that other situational factors and individual differences such as motivation to learn, learning style and personality may influence mathematical learning. However, in England at least, the structure of the mathematical curriculum at the time of participant testing for this research was highly prescriptive, and one could also reasonably assume that there should be few classroom/teacher situational factors that would be heavily influential in differences between schools, given that all children in England had been taught according to this rigid structure. Perhaps difference in abilities at this point could be explained by different teaching competencies, individual differences in the pupils or other external socio-economic factors as mentioned.

5.2 Longitudinal studies

There have been a number of studies that have evaluated the influence of working memory upon mathematical achievement over a period of time. These studies have adopted a variety of approaches, examining typically developing children (Bull et al., 2008; De Smedt, Janssen, et al., 2009) and those children with a mathematical disability or comorbid problems such as a reading disability also (Andersson, 2010). The time frames in which the longitudinal associations have been made also vary from four months up to five years. In a UK based study, Bull et al (2008) found that visuospatial working memory in pre-schoolers predicted performance in mathematics at the end of the third year of primary school. They identified this influence to be evident on of four aspects of maths; problems of simple and complex arithmetic, number sequencing, and graphical representation of data. De Smedt et al (2009) also reported that working memory clearly predicted later mathematics achievement. Their study suggests that the central executive was a unique predictor of performance on both first- and second-grade mathematics. Furthermore they found that there were age-related differences with regard to the contribution of NV-STM and V-STM to mathematics achievement. They put forth that the visuospatial sketchpad was a unique predictor of first-grade, but not second-grade maths, and the phonological loop was evidenced as a unique predictor of second-grade, but not first-grade, mathematics achievement. In a relatively recent study, Geary (2011) agrees that the central executive is a predictor of growth in mathematics, and that NV-STM was uniquely predictive of maths as opposed to measures of V-STM, which were uniquely associated with word reading. It is clear there is some level of agreement in the past literature that central executive tasks can predict mathematics outcomes, when mathematics is measured at a later date (Geary, 2011;

Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Swanson, 2011; Swanson & Kim, 2007; Zheng et al., 2011), but in this field of research many of the studies have been based outside of the UK, and very few have direct associations with the UK mathematics curriculum. Only a small number of studies have longitudinally assessed children in UK education (Bull et al., 2008; Gathercole et al., 2003; Jarvis & Gathercole, 2003).

5.3 Examination of the Relationship between Digit Recall and Mathematics

There has been some reporting of disproportionately strong associations between numerically grounded measures of working memory and mathematical performance (Lehto, 1995) and this evidence was being increasingly supported and reported (Holmes & Adams, 2006; Passolunghi & Siegel, 2001). One possible explanation for this is that working memory and mathematics are linked as the assessments of both involve either number processing or direct access to numerical information. Holmes and Adams (2006) omitted numerically grounded tasks from their study to avoid the results being unduly influenced by this possible relationship. The present study included Digit Recall in order to test this relationship and examine if the exclusion of a numerical measure was necessary and appropriate.

5.4 Aims and Research Questions

The key aims and research questions arising in this chapter are:

1. Only a very small number of studies have systematically examined the unique contributions of the WM components to overall curriculum based mathematical ability (Holmes & Adams, 2006; Holmes et al., 2008; Jarvis & Gathercole, 2003), therefore the primary aim of this study was to assess the different contributions of each working memory component upon UK curricular mathematics.
2. This thesis also considers the longitudinal predictors of UK curricular mathematical attainment, hypothesising that CE-CWM would be most predictive of UK curricular mathematics.

3. A methodological consideration key to research in this field is that WM and mathematics have been shown to be highly related (Lehto, 1995; Passolunghi & Siegel, 2001) and it has been suggested that this is because the measurement of some working memory measures involves either number processing or direct access to numerical information. Therefore the study wanted to ascertain if there was any evidence that digit based working memory tasks are more highly associated with mathematics than non-numerical working memory tasks. This was achieved by assessing the difference in the strength of correlations between the relative tasks.
4. This study also sought to examine the possibility of developmental fractionation of nonverbal STM in a younger sample of children than assessed in the previous literature (Pickering, 2001; Pickering et al., 2001)

To summarise, the present study was designed to examine systematically the contributions of three different components of the working memory model to a curriculum based mathematical test over a three year time period, using measures of working memory that do not involve numerical stimuli.

5.5 Methodology

The methodology for the three-year study was fully described in Chapter 4. To summarise participants were tested on seven working memory measures over two short sessions. Alloway and colleagues devised the computer based working memory battery, the Automated Working Memory Assessment (AWMA) (Alloway, Gathercole, & Pickering, 2004) which now has normative data for children from the age of four for listening recall, as a result a decision was made to use this task in this study. As earlier detailed, the participants in our pilot studies struggled with demands of the AWMA computerised version of the Mazes Memory task, so the pencil and paper version from the WMTB-C (Pickering & Gathercole, 2001) was adopted. The remaining four working memory measures were Word Recall, Nonword Recall, Block Recall and Odd One Out. Digit recall was also measured in order to allow statistical analyses to determine if the reported associations between numerical based working memory tasks and mathematics are robust.

The children undertook the mathematics assessment (age appropriate: Mathematics 5, 6, 7) in a final separate testing session. Working memory raw scores were recorded for the analysis, and Mathematics raw scores were used in order that age could be controlled for in both the mathematics and the WM measures.

5.6 Results

5.6.1 Descriptive Statistics

Descriptive statistics for the children on cognitive measures across the three years are shown in Table 4. Performance measures were not administered during the Reception testing sessions. Mean scores for the working memory tasks identify that the children achieve higher scores on the measures of Verbal STM than the other working memory tests. Scoring on the CE-CWM measures is low, as they are typically more difficult tasks to undertake. Listening Recall scores (verbal CE-CWM) were poorer than Odd One Out (nonverbal CE-CWM), and Mazes Memory scores (NV-STM) worse than the second NV-STM measure, Block Recall. Across all three testing time points it can be noted that the scores on the all of the measures increase as expected, and that the standard deviations are stable. Following examination of the standard error of both the skew and kurtosis, for most of the measures these values indicated reasonable normal distribution. There is a small, but significant negative skew on the NV-STM measures (.78 for Mazes Memory and .81 for Block Recall at Reception year); however as the measures are criterion-referenced and measuring performance (number of items correct), and not norm-referenced, a slight negative skew such as this is not problematic for the data analyses that follow (Brown, 1997). Likewise in the Reception year the listening recall (V-CWM) and Nonword Recall (NV-CWM) measures both suffer from slight kurtosis which relates to how narrowly or widely the data is distributed, and influences the peak of the distribution. Tabchnick and Fidell (2001) suggest that a robust test of the kurtosis is to multiply the standard error of kurtosis by two, and the resulting figure is a guide to the range where the data should fit to show a normal pattern of distribution, however they also recommend that a kurtosis statistic of less than +/- 2.00 means that the data is within adequate parameters. In the cases of both CE-CWM measures this is true *(LR kurtosis = 1.60, OOO kurtosis*

= -1.19); as such the skew and kurtosis data is within the acceptable normal distribution. Overall performance on the Mathematics 5 and 6 tests fell slightly below the mean from the standardised sample, but by Year 2 the results were comparative with the norms of the sample.

Table 4. Descriptive Statistics for WM, Performance measures and Mathematics scores (n=70).

		Reception			Year 1			Year 2		
		Mean	Std. Dev.	Range (Min-Max)	Mean	Std. Dev.	Range (Min-Max)	Mean	Std. Dev.	Range (Min-Max)
Age in Months	**(Age)**	61.01	3.80	14(54-68)	72.74	3.91	15(65-80)	84.63	3.83	14(77-91)
V-STM										
Word Recall	**(WR)**	15.40	3.24	17(7-24)	17.19	3.31	17(8-25)	19.34	3.06	12(13-25)
Nonword Recall	**(NWR)**	12.97	3.14	19(2-21)	14.03	3.26	15(6-21)	15.04	2.37	11(10-21)
CE-CWM										
Listening Recall	**(LR)**	3.17	2.50	8(0-8)	4.90	2.74	11(0-11)	6.86	2.96	15(0-15)
Odd One Out	**(OOO)**	8.96	3.59	15(1-16)	11.67	4.05	16(3-19)	12.06	4.42	21(4-25)
NV-STM										
Mazes Memory	**(MM)**	3.01	2.69	11(0-11)	5.27	3.78	18(0-18)	7.44	4.21	23(0-23)
Block Recall	**(BR)**	10.21	4.05	15(5-20)	12.99	4.39	18(5-23)	16.16	5.23	20(6-26)
Mathematics										
Maths 5/6/7 (Raw)	**(M5, M6, M7)**	14.21	4.28	17(4-21)	16.64	5.24	21(5-26)	17.61	5.51	22(5-27)
Maths 5/6/7 (Standardised)	**(M5ss, M6ss, M7ss)**	91.66	11.53	43(70-113)	96.87	11.78	63(71-134)	98.73	12.14	50(72-122)
Performance Measures										
Object Assembly	**(OA)**	-	-	-	12.57	5.03	21(3-24)	15.41	4.74	18(5-23)
Block Design	**(BD)**	-	-	-	10.07	5.92	30(0-30)	17.11	7.83	35(3-38)

5.6.2 Working Memory Developmental Trajectory

The pattern of increasing levels of performance on working memory tasks in the successive age groups is demonstrated in Figure 3. Consistent with other investigations of cognitive growth (Alloway et al., 2006; Swanson et al., 2008) raw working memory scores were converted to z scores thus scaled to a mean of zero and a standard deviation of 1. It was necessary to do this across the cohort (n=70), so that all parameters were on the same metric, enabling meaningful comparisons for both age and time.

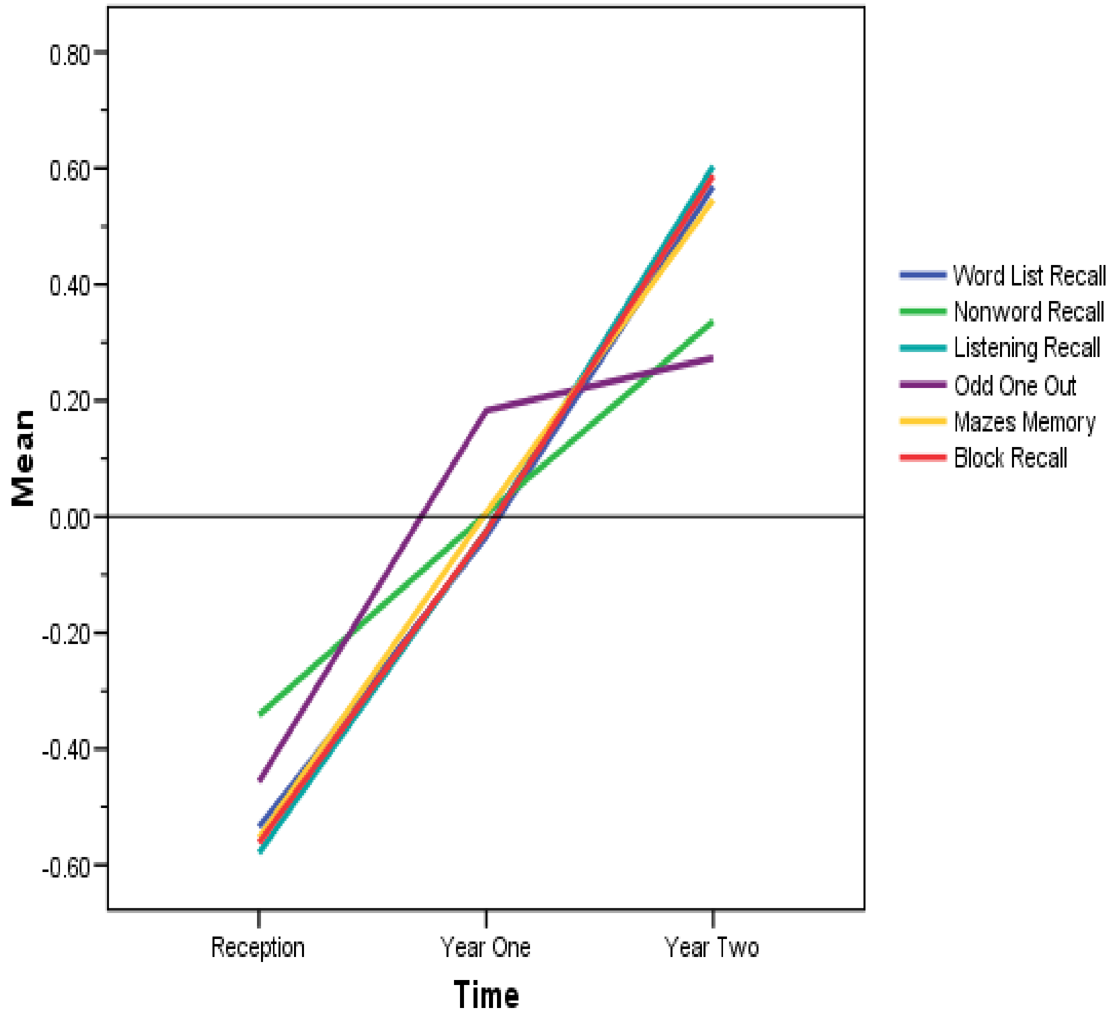

Figure 3. Working memory developmental trajectory

Figure 3 plots mean z scores for each year group *(n=70).* All six tests yielded a broadly similar developmental path, with performance increasing linearly from 5 to 7 years in general. The only marked departure from this profile was observed for Odd One Out, on which scores appear to plateau at age 7. These trajectories are largely comparable with those exhibited in Gathercole et al (2004).

5.6.3 Correlational Analyses – Working Memory and Performance Measures

5.6.4 Reception

5.6.4.1 Within-construct correlations

As anticipated there were significant correlations between the V-STM measures, Word Recall and Nonword Recall *(r=.45, p=<.001).* Alloway et al (2006) find a considerably stronger correlation between these two measures *(r=.62, p=<.0001).* The difference in correlation strength may be explained by the fact that the correlations presented in the Alloway paper (2006) utilise the data from all ages tested (range 4-11yrs) and the coefficients are reported to be slightly inflated due to this large variation in age. There were also significant associations between Listening Recall and Odd One Out *(CE-CWM r=.38, p=<.001)* and between NV-STM measures *(r=.25, p=<.05).* These data are represented in Table 5.

5.6.4.2 Cross-construct correlations

It can be notes that both V-STM measures correlated significantly with listening recall which is the verbal working memory task, but not with Odd One Out, the non-verbal working memory task. Nor did they correlate significantly with either of the NV-STM measures.

Table 5. Correlations between cross construct working memory measures (Mean age 61m, SD 3.80).

		1	2	3	4	5	6
1	**Word Recall**	-	*.51***	*.41***	*.28***	*.33***	*.12*
2	**Nonword Recall**	.45***	-	*.32***	*.11*	*.10*	*-.02*
3	**Listening Recall**	.36***	.27*	-	*.43***	*.34***	*.10*
4	**Odd One Out**	.18	.02	.38***	-	*.38***	*.34***
5	**Mazes Memory**	.17	-.03	.27*	.28*	-	*.38***
6	**Block Recall**	-.05	-.15	.01	.25*	.25*	-

*p=<.05, **p=<.01, ***p=<.001
Partial correlations controlling for age during Reception year in the lower quadrant and zero-order correlations in the upper quadrant.

The NV-STM measures both correlated significantly with Odd One Out, which is the nonverbal working memory task, therefore this would be expected. The expected weak effect size between measures of V-STM and measures of NV-STM indicates consistency with the idea that these WM components are indeed separable, even at this young age.

5.6.5 Year One

5.6.5.1 *Within-construct correlations*

As Table 6 indicates, all six WM measures are significantly inter-correlated at *r=.19* and above *(sig. p=<.05),* and once again the weak effect sizes are evident between measures of V-STM and measures of NV-STM, indicating the likely functional independence of those WM domains.

5.6.5.2 *Cross-construct correlations*

Regarding the performance measure data, which was assessed for the first time at this age grouping, it is shown that aside from Object Assembly and Nonword Recall *(r=.20, p=<.05)* none of the other V-STM and V-WM measures were significantly correlated with the Performance Measures. On the understanding that both performance tasks are of a visual and spatial nature, this may strengthen the idea that verbal short-term memory is indeed quite separable from visual and spatial functions. Given this notion Performance Measures could also reasonably expected to correlate significantly with both of the NV-STM measures, which is largely the case. Block Recall and Block Design do not show a significant intercorrelation, however each of the other correlations is significant.

Table 6. Correlations between cross construct working memory & Performance Measures (Mean age 72.7m, SD 3.91).

		1	2	3	4	5	6	7	8
WM Measures									
1	**Word Recall**	-	*.80***	*.46***	*.44***	*.39***	*.39***	*.26***	*.31***
2	**Nonword Recall**	.76***	-	*.41***	*.49***	*.43***	*.39***	*.30***	*.26**
3	**Listening Recall**	.36***	.32**	-	*.50***	*.29***	*.38***	*.20**	*.40***
4	**Odd One Out**	.32**	.39***	.41***	-	*.37***	*.50***	*.17*	*.46***
5	**Mazes Memory**	.29**	.34**	.19*	.26*	-	*.40***	*.46***	*.57***
6	**Block Recall**	.25*	.26*	.26*	.39***	.30**	-	*.32***	*.24**
Performance Measures									
7	**Object Assembly**	.14	.20*	.10	.05	.39***	.21*	-	*.54***
8	**Block Design**	.16	.12	.30**	.34**	.51***	.08	.48***	-

*p=<.05, **p=<.01, ***p=<.001

Partial correlations controlling for age during Year One in the lower quadrant and zero-order correlations in the upper quadrant.

5.6.6 Year Two

5.6.6.1 *Within-construct correlations*

In this year grouping the data shows that the measures of V-STM correlate at *r=.61, p=<.001*, the central executive measures were also strongly correlated (*r=.58, p=<.001*), however the two measures of NV-STM failed to correlate (Table 7). This is somewhat unusual and may provide indications that there is a developmental shift, or it could be indicative of dissociation between the two measures (Pickering, 2001; Pickering et al., 2001; Pickering et al., 1998).

5.6.6.2 *Cross –construct correlations*

Both V-WM and NV-WM tasks were significantly intercorrelated with both Performance Measures. Mazes Memory and Block Recall each achieved correlational significance with Object Assembly but not with Block Design. Following the data shown in Table 7 it is also apparent that the patterns appear to have altered in which of the Performance Measures correlate with the NV-STM measures. Previously Mazes Memory had correlated with both Performance Measures, and Block Recall had correlated with Object Assembly, however in these results Block Recall switches, and now correlates with Block Design, although the effect size is weak. Finally, Object Assembly and Block Design correlated at *r=.46, p=<.001.*

Table 7. Correlations between cross construct working memory & Performance Measures (mean age 84.m, SD 3.83).

		1	2	3	4	5	6	7	8
WM Measures									
1	**Word Recall**	-	*.64***	*.41***	*.41***	*.23***	*.40***	*.16*	*.18*
2	**Nonword Recall**	.61***	-	*.43***	*.39***	*.21**	*.42***	*.09*	*.12*
3	**Listening Recall**	.37***	.39***	-	*.61***	*.08*	*.52***	*.26**	*.39**
4	**Odd One Out**	.34**	.34**	.58***	-	*.21**	*.55***	*.27**	*.44***
5	**Mazes Memory**	.17	.16	.02	.12	-	*.14*	*.18*	*.44***
6	**Block Recall**	.35**	.39***	.49***	.51***	.08	-	*.20**	*.32***
Performance Measures									
7	**Object Assembly**	.10	.03	.21*	.20*	.12	.14	-	*.49***
8	**Block Design**	.13	.08	.36***	.40***	.21*	.29**	.46***	-

*p=<.05, **p=<.01, ***p=<.001
Partial correlations controlling for age during Year Two in the lower quadrant and zero-order correlations in the upper quadrant.

5.6.7 Working memory and mathematics correlations

The correlation coefficients reported in Table 8 were calculated to examine the associations between the working memory tasks administered and mathematics over the three year period including Digit Recall. Digit Recall is now included for the purpose of examining the relationship between a numerically based working memory measure and mathematics. These data are shown as zero-order correlations, as age related variance will be statistically controlled for in more detailed regression analyses later in the chapter.

Nonword Recall from the Reception year failed to correlate significantly with Mathematics 5 test scores for this sample, but at both Year One and Two Nonword Recall did correlate significantly with the relevant mathematics outcome. All other working memory measures correlated significantly with the mathematics outcome, with strong effect sizes, ranging between *r=.30, p=<.005 and r=.68, p=.001*.

This table also identified the strong correlational association between Digit Recall and mathematics, particularly in the Reception and Year One groups. A similarly strong relationship is evident between Odd One Out and mathematics at each age grouping. However, as the Odd One Out task has no numerical bearing, and is a visuospatial working memory task these results suggest that it could be viewed as a more "pure" measure when considering working memory and relationships with mathematics. This is because it does not have the added numerical loading that Digit Recall naturally has. The issues with the relationship between Digit Recall and mathematics will be discussed in more detail in Section 5.7.

Table 8. Zero order correlations between working memory measures and Mathematics (5-14 series) over a three year period.

	V-STM									CE-CWM						NV-STM					
	DR			WR			NWR			LR			OOO			MM			BR		
	R	Y1	Y2	R	Y1	Y2	R	Y1	Y2	R	Y1	Y2	R	Y1	Y2	R	Y1	Y2	R	Y1	Y2
Maths 5	.65	-	-	.30*	-	-	.09(ns)	-	-	.39*	.-	-	.62	-	.-	.48	-	-	.46	-	-
Maths 6	.65	.68	-	.47	.63	-	.17(ns)	.69	.-	.46	.64	-	.58	.66	.-	.46	.50	-	.31*	.49	-
Maths 7	.58	.41	.34*	.35*	.48	.35	.09(ns)	.54	.39	.35*	.64	.59	.53	.39	.57	.46	.48	.33*	.33*	.47	.41

* p<.005, (ns) = non-significant, all other *rs* p=<.001

5.7 Digit recall

Digit recall data had been collected as an original part of this research as initially it was to be included in the analyses. However early evidence of excessively strong associations between Digit Recall and mathematical performance (Lehto, 1995) was being increasingly supported and reported (e.g. Holmes & Adams, 2006; Passolunghi & Siegel, 2001). A decision was made to omit Digit Recall from the hierarchical regression analyses due to these reported associations as in Holmes and Adams (2006) to avoid the possibility of over inflating the hypothesised associations between working memory and maths. However, since making this decision some data has emerged suggesting that there is no significant difference in the associations between numerical and non-numerical V-STM tasks and mathematics (Alloway, 2007). Given the conflicting opinions on this matter it was considered useful to be able to assess these data in this cohort study in order to validate the previous research. To facilitate this assessment the difference in the strength of correlations, if any, between numerical V-STM tasks (Digit Recall) and mathematics scores; and word-based V-STM tasks (Word Recall, Nonword Recall) and mathematics scores the difference between correlation coefficients was calculated based on the value of the coefficients and the sample size (Cohen & Cohen, 1983). In the Reception year there was a significant difference between the effects of Digit Recall and Word Recall on maths outcome *[$t(69)=5.91$, $p=<.05$],* and a significant difference observed between correlations of Nonword Recall and Digit Recall and mathematics scores *[$t(69)= 6.09$, $p=<.05$].*

However, in Year One these data show that there was no significant difference between the effects of Digit Recall and Word Recall on maths performance *[$t(69)= -.50$, ns],* nor between the effects of Nonword Recall and Digit Recall on maths performance *[$t(69)= -1.03$, ns].* In the third and final testing phase there was again a significant difference evident between the correlations for numerical V-STM and Word Recall *[$t(69)= 2.33$, $p=<.05$]* and a significant difference between the effects of Digit Recall (numerical V-STM) and Nonword Recall (non-numerical V-STM) upon mathematics performance *[$t(69)= 3.12$, $p=<.05$].*

Given Alloway's (2007) findings, where the author finds no statistical difference between the numerical and non-numerical verbal STM tasks there is a speculative possibility that the non-significant result achieved could be a product of the sample in the Alloway study, who were children with developmental communication disorder, or perhaps a product of age as the children were considerably older in the Alloway paper (age range 6-11 years).

There is a clear anomaly in this present study between the data from the first and last time points where there is evidence of a significant difference between numerical and non-numerical verbal short-term memory tasks upon mathematics performance, and the Year One data where no significant difference is apparent. Given that there is still no definitive clarity on this issue it was considered that the decision to remove numerically grounded verbal short-term memory tasks from the study remains justified.

5.8 Regression Analyses

Regression is a statistical tool used to investigate relationships between variables, and in particular assess the predictive value of one variable, or set of variables over and above the other specified variables which allows us to make rational decisions about the effect of adding additional information on the accuracy of prediction. The ratio of cases to independent variables should ideally be 20:1, and should certainly be no lower than 5:1, in these data the case to independent variable ratio was 14:1.

In all of the subsequent tables R^2 indicates the total variance predicted by the regression model in question and the R^2 *Change* statistic ($R^2\Delta$) shows the variance uniquely contributed by the predictor variables entered at the last step of the regression equation.

To facilitate this more detailed analysis of these relationships a succession of fixed-order hierarchical regression analyses were performed (Table 9). Hierarchical regression models were used as this allowed the input of predictor variables into the equation in a specific order based upon past research in this field. Therefore, these models assessed the amount of unique variance in mathematics scores predicted by each of the individual working memory measures after statistically controlling for age related variance and any variance pertaining to the Performance Measures.

5.8.1.1 Reception

In this first series of hierarchical regression analyses Model 1: Reception demonstrated that the WM model accounted for 28% of the unique variance in mathematics scores at Reception age *(R^2 Δ=.28, p=<.0001, ANOVA [f (7, 62) =11.21, p=<.0001])* after eliminating any age related variance from the model (Table 9), with a significant beta value identified for Odd One Out (NV-WM). The adjusted R^2 figure indicates how well the findings can be generalised, and the minimal shrinkage from R^2 *=.56*, to *adj* R^2 *=.51 is .05*, and as such if this model were derived from the wider population it could be expected to account for 5% less variance in the outcome measure (Maths 5). Durbin-Watson[3] test was checked and within acceptable parameters *(2.27).*

The beta values tell us to what degree each predictor variable affects the outcome measure if all other predictors were held constant. There are two significant beta values indicating that both Age and Odd One Out (NV-WM) are significant independent predictors of Mathematics 5. The standardised beta coefficient *(*β*)* is measured in standard units, meaning that they are directly comparable with one another. The β for age and Odd One Out are *.29* and *.39* respectively. This indicates that in this model Odd One Out is slightly more important than Age in predicting scores on Mathematics 5. In real terms one would expect to see an increase of 1.40 in scores on Mathematics 5 with every standard deviation increment on Odd One Out (and an increase of 1.23 in Mathematics 5 scores for Age), the caveat of course being that these interpretations only hold true if all other predictor variables remain the same. Problems of multicollinearity were also checked for using the variance inflation factor (VIF) which quantifies the severity of multicollinearity, providing an index that measures how much the variance (the square of the estimate's standard deviation) of an estimated regression coefficient is increased because of

3 Durbin-Watson test is a test to the assumption of independence of the residuals. The test statistic is between 0 and 4 and a value of 2 means that the residuals are uncorrelated. As a general rule of thumb a value of between 1 and 3 will not give rise to cause for concern.

collinearity. At 1.57, the largest VIF was well below 5, and the average VIF was under 1.49, similarly the tolerance data are all well within acceptable boundaries (all greater than 0.1).

One case was identified as an outlier, however scrutiny of the casewise diagnostics (Cooks' Distance: none greater than1; average leverage = 0.1) indicates that this outlier is not having an undue effect upon the model and that our sample appears to conform to what would be expected for a fairly accurate model.

5.8.1.2 Year One

In Year One (Model 2) working memory significantly accounted for 36% of the variance in mathematics scores *(R^2 Δ=.36, p=<.001; ANOVA [F (9, 60) =17.66, p=<.001])* after eliminating both age related and non-verbal performance related variance from the model (Table 9). The adjusted R^2 figure shows that if this model were resultant from the wider population it could be expected to account for 4% less variance in Mathematics 6 (difference between R^2 *=.73* and *adj* R^2 *=.69 is .04*) again Durbin-Watson test was checked and inside satisfactory bounds *(1.80).*

The β values inform us that both Listening Recall (V-WM) and Nonword Recall (V-STM) are both significant independent predictors (β*=.38, and .26* respectively), with Odd One Out (NV-WM) approaching significance levels *(β=.18, p=<.06).* There were no issues of multicollinearity detected.

5.8.1.3 Year Two

In the final year (Year Two) working memory contributed 17% *(p=<.005)* of the unique variance in Mathematics 7 scoring (Model 3). The V-WM measure Listening Recall contributed significant independent variance to maths performance *(β=.30, p=<.01)* even after all the other variables had been partialled out. As shown none of the other WM variables contributed significant variance. From the Performance Measures Block Design was also a significant unique predictor with β*=.30, p=<.01*. No concerns about multicollinearity were found.

The regression analyses find that at no point is either of the measures of nonverbal short-term memory significantly predicting mathematics test performance as a whole.

Table 9. Hierarchical regression models predicting mathematics performance with WM measures, controlling for age.

	Model 1: Reception Regressor: Mathematics 5 Raw Score							Model 2: Year One Regressor: Mathematics 6 Raw Score							Model 3: Year Two Regressor: Mathematics 7 Raw Score						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1 **Age in Months**	.28	.27	.28	26.23*	.33	.12	.29*	.26	.25	.26	23.79***	.13	.11	.10	.13	.11	.13	9.69**	.14	.14	.10
Step 2 **Performance Measures**	-	-	-	-	-	-	-	.37	.34	.11	5.77**				.40	.38	.28	15.32***			
Object Assembly					-	-	-					-.09	.09	-.09					.09	.12	.09
Block Design					-	-	-					.15	.09	.17					.21	.08	.30**
Step 3 **Working Memory**	.56	.51	.28	6.56*				.73	.69	.36	13.00***				.58	.51	.17	4.07**			
Word Recall					.01	.14	.01					-.01	.19	-.01					-.07	.21	-.04
Nonword					-.12	.14	-.09					.61	.20	.38*					.34	.27	.15
Listening Recall					.22	.18	.13					.50	.16	.26*					.55	.22	.30*
Odd One Out					.47	.12	.39*					.24	.13	.18a					.17	.15	.14
Mazes Memory					.16	.17	.10					.11	.13	.08					.18	.12	.14
Block Recall					.17	.10	.16					.08	.11	.06					-.03	.14	-.02
ANOVA	ANOVA [f(7,62)=11.21, p=<.0001]							ANOVA [f(9,60)=17.66, p=<.001]							ANOVA [f(9,60)=9.03, p=<.001]						

Mathematics 5 * p=<.0001; Performance measures not assessed
Mathematics 6 * p=<.01, **p=.005, ***p=<.001, a p=.06
Mathematics 7 * p=.05, **p=.005, ***p=<.001, a p=.06

5.9 Predicting Overall Mathematics using WM measures at School Entry

A final regression analysis was calculated to examine the predictive value of working memory when assessed at Reception age upon mathematics performance 2 years subsequent. Table 10 indicates the statistical results for this analysis. What is evident is that concurrent with the annual analyses above, the consistent predictive factor emerging from the WM variables is central executive, in particular Odd One Out (NV-WM). This is after any unique variance accounted for by Age at Y2 and Performance Measures at Y2 has been removed from the statistical equation.

Table 10. Hierarchical regression model predicting mathematics performance at Year 2 with WM measures measured 2 years previous, controlling for age and Performance Measures.

	Model 4: Regressor: Mathematics 7 Raw Score						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1	.13	.11	.13	9.69*	.18	.16	.12
Age in Months at Y2							
Step 2	.40	.38	.40	14.80**			
Performance Measures							
Object Assembly Y2					.09	.12	.08
Block Design Y2					.28	.08	.40**
Step 3	.54	.47	.14	2.93*			
Working Memory (R)							
Word Recall					.29	.19	.01
Nonword					-.26	.19	-.09
Listening Recall					.15	.24	.13
Odd One Out					.50	.16	.29***
Mazes Memory					.01	.24	.10
Block Recall					.03	.14	.16
ANOVA	ANOVA [f(9,60)=7.75, p=<.0001]						

*p=<.01, **p=.001

The R^2 change statistic *(R^2 Δ=.14)* is showing that even after all of these other key factors have been taken into consideration, WM measured at Reception age is still contributing 14% of the unique variance in scoring on the mathematics test when mathematics is assessed 2 years later. While this is a relatively small amount, it is still highly significant.

5.10 Discussion

A growing body of work is suggesting that working memory is related to performance in mathematics from an early age and right through the school career (e.g., Bull et al., 2008; Holmes & Adams, 2006; Holmes et al., 2008; Passolunghi & Cornoldi, 2008; Simmons et al., 2012). Each of these studies has been very informative; however there have been some cross-study issues that have resulted in somewhat inconsistent conclusions. Examples of issues that may have influenced the conflicting results might be; the use of a wide variety of both mathematical and working memory tasks that are not necessarily standardised against one another, the use of numerical working memory measures, the use of only one measure per working memory domain, and the varying age ranges assessed. In the present study in order to attempt to combat some of these issues the number of working memory measures used was increased to two per WM component, and refrained from the use of numerically grounded tasks. A further strength of this study was that a cohort of children was followed over a three year period during their early schooling. This allowed the study to elucidate further the specific elements of working memory that were thought would be involved expressly in Calculation performance over a three year period.

Overall the results across the three year period further substantiate the close links evidenced between the Baddeley and Hitch (1974a) model of working memory and performance on a curriculum based mathematics test (Bull et al., 2008; Holmes & Adams, 2006). The cross-sectional results confirmed that working memory is accounting for between 17% and 36% of the unique variance in mathematics raw scores after age related variance has been partialled out. Agreement with the Rasmussen and Bisanz study (2005) is noted insofar as WM is predicting a significant amount of unique variance at this specific age range. This finding also lends support to previous evidence that suggests that there is a relationship between working memory and National Curriculum performance given that the administered mathematics test was heavily influenced by the teaching of the National Curriculum (Gathercole & Pickering, 2000b; Holmes & Adams, 2006; Jarvis & Gathercole, 2003). Moreover these data extend previous findings by applying the working memory theoretical principles to curriculum based mathematics at a younger age range than measured in the previously cited studies.

5.10.1 Reception

While NV-STM and CE-CWM both significantly correlate with Mathematics 5 raw scores, the in-depth statistical analyses demonstrated that at this age CE-CWM is the only significant independent predictor variable for performance on the mathematics test. The CE-CWM finding in this phase of testing supports some previous studies (Bull et al., 2008; Holmes & Adams, 2006), and corroborates the idea that carrying out mathematical operations is likely to involve executive functions such as inhibition, task switching, strategy adoption and updating (see also Bull et al., 1999). However, while support for (Bull et al., 2008; Bull et al., 1999) is proffered, direct comparisons cannot be immediately drawn as the tests utilised as measures of central executive/executive function in the Bull et al studies were different from those in the present work. In relation to the functional similarity of executive function and central executive tasks Lehto (1996) reports that Wisconsin Card Sorting Task (WCST), the Tower Of Hanoi, and the Global Search Task each appear to tax different aspects of executive function, with only the WCST being dependent on working memory. As such it is thought that more meaningful comparisons may be drawn from studies where the WCST was used as a measure of executive function/working memory (Bull et al., 1999).

Bull and colleagues also reported that Corsi Block was not related to mathematical ability (at mean age 7.3y), and with this measure a more meaningful comparison can be drawn, as the Block Recall task is a computerised version of Corsi block task. In this present study, the Reception data agree with Bull and colleagues showing that at this age grouping Block Recall (as a measure of NV-STM) is not a significant predictor of mathematics (as a whole) even though Block Recall and Mathematics 5 do correlate strongly.

5.10.2 Year One

At Year One (mean age 72.7 month, SD 3.91) as with the results from the Reception year at school, these data advocate links between the Baddeley and Hitch (1974a) tripartite model of working memory and performance on this curriculum based mathematics test, with the working memory measures accounting for 36% of the unique variance in overall mathematics raw scores. This figure is after taking statistical account of both age and the newly introduced Performance Measures. All six WM measures correlate significantly with Mathematics 6 raw scores, yet the in-depth statistical procedures indicate that at this age the verbal working memory measure and

V-STM are both significant independent predictors of performance on the mathematics assessment after all the other variables have been accounted for. Similarly Gathercole et al (2006) also found tests of central executive and verbal short-term memory, but not non-verbal short-term memory to be indicative of mathematics performance. Additionally Holmes and Adams (2006) established that verbal short-term memory skills were related to mental arithmetic, but not to other mathematics skills measured in 8-10 year olds.

Once again, at this age range as with Bull et al (1999) no evidence is found to suggest that nonverbal short-term memory is contributing to predicting the Mathematics 6 outcome.

5.10.3 Year Two

The pattern of the Year 2 data follows a similar blueprint to the previous two years with working memory as a whole being a significant predictor of Mathematics 7. Also evident is that a central executive task is emerging as a consistent unique predictor of performance on this curriculum based mathematics assessment. However at this time point the significant independent predictor was Listening Recall (V-WM).

As with Reception and Year One, the nonverbal short-term memory measures showed no influence over performance on the mathematics task. It was also noted that two nonverbal short-term memory variables failed to intercorrelate at Year 2 (Mazes Memory and Block Recall) but both did correlate with Mathematics 7. There are several feasible explanations for this anomalous data pattern. Two ideas that stand out are that this age grouping may indicate a developmental fractionation time point (Alloway et al., 2006; Hitch, 1990). A second explanation may be that the difference between the Mazes Memory static presentation format and the Block Recall dynamic presentation format may prove to be the key factor in the disparity between the two tasks. Pickering and colleagues (Pickering, 2001; Pickering et al., 2001) have reported evidence of a developmental dissociation in performance on static and dynamic versions of the matrices task suggesting that it may not be the visual and spatial properties of two tests used in their study of visuospatial memory (Corsi blocks and the visual pattern test) but the static and dynamic nature of the tasks that taps different subcomponents of this memory system. It is possible, therefore, that NV-STM may comprise of separable components for dealing with visuospatial information in the form of static patterns and paths of movement. It is tentatively proposed that this may only become evident at around the age of 6 to 7 years old, as previously in this study the NV-STM measures correlated adequately for tasks reputed to be measuring the same construct.

5.10.4 Predicting Later Mathematics with Early Working Memory

Some studies have claimed that intelligence tests are a reliable index to predict later scholastic attainment (Colom & Flores-Mendoza, 2007; Stanovich, Cunningham, & Feeman, 1984). Recent works however, have strongly intimated that working memory represents a dissociable cognitive skill from intelligence, with unique links to learning outcomes, and it is also relatively culture free (Alloway, 2009; Fischer, 2008). It is arguably too soon into the theoretical understanding and debate about this topic to adopt the working memory approach to replace intelligence testing, but it is proving over and over to be a robust finding that working memory can predict later school progress (Gathercole et al., 2003; Gathercole & Pickering, 2000b; Gathercole, Pickering, Knight, et al., 2004; Holmes & Adams, 2006; Jarvis & Gathercole, 2003; Mayberry & Do, 2003; St Clair-Thompson & Gathercole, 2006). This present study has demonstrated that working memory measured upon school entry can uniquely predict 14% of variance in overall mathematics scores at age 7. This figure is obtained after the variance pertaining to both age and Performance Measures has been statistically removed.

Other studies such as Passolunghi et al (2007) have also identified WM as a predictor of maths over a period of time. In their instance mathematics was measured four months post working memory testing. In a similar type of study Noël, Seron and Trovarelli (2004) provided evidence that phonological loop capacity was indicative of later mathematics ability, with particular reference to addition skills and strategies in first graders, who are roughly age comparable with the present cohort. Clearly these studies link well with our finding that working memory significantly predicts a portion of later overall curricular mathematics ability. Interestingly there is a minor incongruity between the present study and the Noël et al research in that no strong evidence is found that verbal short-term memory is significantly predictive of later maths ability. In the Noël study the emphasis was placed upon addition and addition strategies rather than general mathematical skills. Therefore at this stage this study cannot discount the notion that phonological processing may well be significantly involved in more “specific” or separable mathematics abilities which will be discussed in later chapters.

5.11 Chapter Summary

1. The aim of this chapter was to analyse the relationship between working memory as a whole theoretical concept and children’s mathematical performance over a three year period. Further aims were to examine the reported excessively strong relationship between Digit Recall and mathematical performance, and to discover if working memory was a good predictor of mathematics attainment when mathematics was measured 3 years post WM testing.
2. Working memory significantly predicts variance in overall mathematics at each time point within the study. Working memory also significantly predicts mathematics performance over time after both Performance Measures and age had been statistically accounted for.
3. However at each age range different aspects of WM are contributing significantly to the variance in scores on the curriculum based mathematics test. It is also apparent that central executive measures are emerging as the better predictor of mathematics in the year-on-year analyses. At Reception a visual measure of working memory is the only significant independent predictor, At Y1 a V-STM measure, and both CE-CWM measures are significantly predictive. At Y2 the verbal WM measure emerges as the lone significant independent predictor variable.
4. These findings extend previous findings to suggest that working memory assessments may be useful early indicators of performance on curricular based mathematics tasks.
5. Supplementary evidence is provided that Digit Recall has a strong association with mathematical tasks. It is acknowledged that this data is not wholly conclusive due to the

anomaly at the Year One grouping, but given the overly strong relationship at two of the three time points it is suggested that numerically based working memory tasks should be avoided when addressing working memory and mathematics.

6. Additionally there is a very cautious suggestion that at Year Two there seems to be some fractionation within the NV-STM measures.

As mentioned in the early part of this thesis mathematics is an umbrella term for a very wide range of skills and competencies. This mathematics test is measuring an assortment of skills such as addition, subtraction, word problem solving, simple weights and measures, simple height differences, as well as properties of number, distance and shape to name but a few. In order to unpack this “mush” the next four chapters each take a core competency (known as a strand) as identified by the National Curriculum/National Numeracy Strategy/Primary National Strategy and examine and discuss the effects of working memory on each of these strands in turn.

Chapter Six

6 Working Memory and the Number Strand

As already discussed in Chapter 3, mathematics under the UK curricular structure can be broken down into several smaller strands. This section of the thesis is concerned with examining the underlying working memory components that may contribute to performance on early number skills, or more specifically the curriculum strand, Number. Number is the most basic of the four strands, but arguably the most important as it provides the foundations for, and establishes the principles of understanding number, counting, the properties of number and number sequences, and includes negative numbers.

6.1 The Number Strand

In order to examine the effects of working memory on mathematical performance in far greater depth the Mathematics 5-7 tests were deconstructed into four key strands, each of which was discussed in Chapter 3. However, to summarise about the Number Strand, at this age range the administered tests can be expanded from a simple raw score to four specific strands. These strands are constrained by common themes and attainments within a specified grouping and learning objectives for the Number Strand can be found in Appendix A. The National Numeracy Strategy's (DfEE, 1999) basic identification of the core competencies to be obtained within the Number Strand is:

- Counting
- Properties of numbers and number sequences, including negative numbers, place value and ordering, including reading and writing numbers
- Estimating and rounding
- Fractions, decimals and percentages, and their equivalence; ratio and proportion

Therefore Number should provide the early foundations for mathematical understanding and help to determine principles of understanding number, counting, columnar structure- such as units, ten, and hundreds, as well as the properties of number and number sequences, including negative numbers and mathematical symbols. Consequently this section is devoted to the principles of working memory that may be underpinning the understanding of basic concepts of number and numerical symbols, quantities, numerosities, columns, counting and such like. This likely involves working memory in its capacity as a temporal storage area for information as it is committed to long-term memory. It is not until a child has mastered the aforementioned basic number principles that teachers can introduce mathematics that requires processing of these facts to be made. Even in the simplest terms a child would have to understand terms such as "more" and "less" and "bigger" "smaller" to be able to make judgements about the magnitudes of an array of items.

This chapter is a difficult one to adequately substantiate with relevant prior research relating to both the working memory model and the UK curriculum. To the best of the author's knowledge there are no studies that preceded the research in this chapter in terms of the Number Strand within the UK mathematics curriculum. However, there are two avenues of research that have influenced the thinking in this chapter.

Holmes and Adams (2006) research with children aged 7 – 10 years old, focussed on curricular based mathematics and the influence of the model of working memory, however Holmes and Adams did not centre this research specifically on the "strands" in as much detail as this present thesis. Whilst acknowledging the previous evidence there is an argument that generalising about working memory abilities from older child populations to younger, developing children will not provide an accurate picture of the topic at the fore.

The second avenue of interest is of studies that have examined numerosity and number sense in young children. The ability to generalise from these studies to the present one however are also limited insofar as working memory is not a key feature of a considerable body of the work in this field. Nevertheless from this related research it is thought that this study may be able to identify areas of number sense and numerosity that may be influenced or mediated by the key cognitive function of working memory.

6.2 Involvement of the Working Memory Model in the Number Strand

The involvement of working memory in general mathematical skills has been extensively discussed throughout this thesis with regard to both the previous literature (Introduction - Chapter 2) and its context with the overall mathematics ability that has been focussed upon in this thesis (Chapter 5). From this discussion it is understood that working memory has an influence in general (Adams & Hitch, 1998; Bull & Espy, 2006; Bull et al., 1999; Geary et al., 2004; Huttenlocher, Jordan, & Levine, 1994; Imbo & Vandierendonck, 2007b; Pennington, 2006; Pennington & Willis, 2004, 2006) and some specific mathematics abilities including addition, subtraction, multiplication and problem solving (Adams & Hitch, 1997; Geary et al., 1987; Mabbott & Bisanz, 2003; Ostad, 1998; Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001), but exactly how the working memory model relates to the UK mathematics curriculum, and in particular the individual mathematics strands is not apparent. A clearer picture is evident when considering some specific maths competencies, particularly those such as addition and subtraction (Adams & Hitch, 1997; Imbo & Vandierendonck, 2007b), but in terms of the basic cognitive principles underpinning mathematics as identified by the Number Strand and curricular mathematics very little is known.

6.2.1 Associations between working memory and Number

Holmes and Adams (2006) tested UK curricular “Number & Algebra” which is primarily number knowledge and counting, but also includes understanding of the four key number operations (add, subtract, multiply, and divide), recognition of number patterns and sequences, and dealing with fractions and decimals, as well as using the related vocabulary to solve problems. Under this curricular structure, and as a consequence of the age of participants the “Number & Algebra” strand in the Holmes and Adams study goes much deeper into more complex processing aspects of Number than the way it is defined by the current study; however this research is the closest to a precursor that is known, and deriving some information from this research is useful. From the

Holmes and Adams paper, first considering correlates it is noted that “Number & Algebra” correlates with NV-STM and CE-CWM after age was statistically controlled for. Holmes and Adams subsequent regression analyses identified that their VSSP (NV-STM) model accounted for 3% of the variance in performance on this Strand, and their CE model accounted for 12% of the variance in overall scores on this Strand. The Holmes and Adams data substantiates previous general findings that the central executive is an important predictor of children’s mathematics in children aged 7-10 years old (Bull et al., 1999; Bull & Scerif, 2001). Across both age groups, CE predicted a significant amount of unique variance on “Number and Algebra”. However Holmes and Adams performed principal components analysis that suggested that tasks measuring central executive loaded on both the WM and mathematics factors, potentially indicating that the CE measure in the study is interrelated to a more general resource such as intelligence (Fry & Hale, 2000).

Approximation and number transcoding are both important skills for children to master and these competencies would fall under the remit of the Number Strand. Working memory has been demonstrated to have an impact upon approximation in 7 year olds and pre-schoolers respectively (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; Xenidou-Dervou, van Lieshout, & van der Schoot, 2013) and Camos (2008) showed a consistent relationship among error rates, working memory capacity, and the quantity of rules in a study with second grade children, where children with a low working memory span performed poorly on all these aspects of transcoding. A subsequent study by Moura et al (2013) gave evidence that the influence of working memory on number transcoding is somewhat selective, with influence of working memory being stronger for effects that reflect the complexity of Arabic numerals and that involve “online” manipulations of numerical units. Whilst these studies are not directly referencing the Number Strand, or indeed the UK curriculum, this is indicative that working memory is influential in key aspects of development of number.

Geary (1993) discussed the importance of nonverbal short-term memory in early number competencies and asserted a specific subtype of MD characterised by visuospatial deficits. The sorts of mathematics deficits arising from VSSP problems were issues such as column misalignment and failure to handle simple place value. Both of these problems are elements clearly specified in the Number Strand (DfEE, 1999, n.d.-b). Furthermore, Mclean and Hitch (1999) also verified that children with mathematics deficits (aged 9 years old) do indeed perform significantly more poorly on measures of visuospatial sketchpad synchronous with the ideas put forward by Geary .

With reference to typically developing children, once youngsters attend formal preschool and primary education they are often helped to represent number using external tokens, such as building blocks, or toys (DfES, 2001). This is in part to make learning a more fun and engaging pursuit, however there is also evidence that the use of concrete material in learning aids effective learning (Ball, 1992). Crucially the DfES advise that concrete visual and tactile support in primary mathematics is an effective way to aid learning in children with dyslexia or dyscalculia who require extra learning support (DfES, 2001). The use of concrete materials as aids to memory might assist children by means of representing number physically and thus reducing the working memory resources necessary to represent that data in short-term memory, however it has been shown in some research that children with poor working memory tend not to use aids (Gathercole & Alloway, 2004). Additionally, in the absence of concrete visual and/or tactile support, it can be noted that the human body is a very efficient and natural means for supporting the physical representations and development of early number skills, and children will unsurprisingly utilise parts of the body to facilitate mathematics understanding and latterly, mathematical processing (Hunting, 2003).

In considering a role for complex working memory in the Number Strand, CE-CWM would typically be identified as being involved in higher order mathematical operations where simultaneous storage and processing is necessary, such as addition or multiplication, and far less so in the early lower order mathematical learning. However there is a suggestion that CE-CWM has a role to play in the commitment to, and retrieval of, early number facts to LTM (Kaufmann, 2002; Zuber, Pixner, Moeller, & Nuerk, 2009). These cognitive theorists have driven two key hypotheses regarding the starting place for a CE-CWM deficit in mathematics, suggesting two potential forms of retrieval deficit. One idea involves an uncomplicated deficit in the ability to retrieve correct number facts from a semantics based long-term memory archive. The second idea posits that the deficits result from disruption to the retrieval process brought about by problems arising from the functioning of inhibitory mechanisms. By way of an example, think about solving a simple addition problem such as 2 + 4. Children with a CE-CWM deficit might be inclined to retrieve 3 or 5 in place of 2 or 4, and sum the incorrect number fact; or they could retrieve both 3 and 5. The rationale being, that these numbers are the next numbers in the counting sequence and thus closely associated with the counting string, and as such inhibiting the retrieval of them is likely to be more difficult.

6.3 Numerosity/Number Sense & Approximate Number System

There has been some recent evidence that makes the claim that number sense is a powerful predictor of later mathematics outcomes (Jordan, Kaplan, Ramineni, & Locuniak, 2010; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005).

Number sense can be described as,

> *"the general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations" (Reys et al., 1999 p.61.).*

Also as

> *…. "a short-hand for our ability to quickly understand, approximate, and manipulate numerical quantities" (Dehaene, 2001).*

Much of the early number sense literature has not examined working memory experimentally, whereas the recent work by Nancy Jordan and colleagues has taken working memory into account; however working memory is not the primary focus driving forth their research. They have first and foremost written about the importance of number sense in contributing towards and predicting mathematical achievements in young American children (Jordan, Glutting, & Ramineni, 2010; Jordan, Glutting, Ramineni, & Dowker, 2008; Jordan, Glutting, Ramineni, & Watkins, 2010; Locuniak & Jordan, 2008). Jordan and colleagues argue that they have developed a reliable measure of Number Sense (Number Sense Assessment Tool), and this 33 item instrument measures counting and number recognition, number knowledge, and number operations (see Appendix A in Jordan et al., 2008). These studies contend that number sense is a powerful tool in the ability to predict subsequent mathematics achievement. However it is my belief that there is a potential confound with the Number Sense Assessment Tool. The test contains a number of arithmetic questions that require the child to calculate a response using mathematical operations. Calculation tasks go beyond the core definitions of number sense and numerosity and therefore there is an argument that the Number Sense Assessment Tool is in part at least, measuring calculation skills as well as number sense. In this regard I believe that what is being evidenced here is early mathematics predicting later mathematics (see also De Smedt, Verschaffel, & Ghesquière, 2009; Geary et al., 1999; Jordan, Glutting, & Ramineni, 2010; Jordan, Glutting, Ramineni, et al., 2010; Jordan et al., 2013; Jordan, Kaplan, et al., 2010; Locuniak & Jordan, 2008), and while it is useful to know with certainty that this is a key predictor variable it is also important to try to understand any other domain general cognitive precursors to early numerical competencies .

A study conducted by Krajewski and Schneider (2009b) has highlighted the importance of the previous mathematical knowledge experienced by a child (preschool) in the development of their mathematical skills. Typically a child will begin to learn counting by reciting the number words around the age of 2, so clearly the expectation is that most children do start their school life with some, albeit small prior knowledge of number. In this study the authors suggest that a likely scenario is that phonological processes are more heavily involved in this early math, as the child will not be discriminating quantities, merely employing recitation skills. This study makes clear that phonological awareness may have an important role to play in the acquisition and automation of the number word sequence but they found that has a negligible direct influence on higher order quantity (those number skills that reflect a conceptual understanding of the quantity to number word linkage (c.f.Krajewski & Schneider, 2009a)). Krajewski and Schneider also indicate that both phonological awareness and VSSP measured in kindergarten directly influenced math-specific precursor variables assessed a few months later (refer to Fig.4. for the three levels of early mathematical precursors). They also identified moderate indirect effects of these two variables on mathematical performance in the subsequent formal schooling (up to Grade 3, age range 8yrs to 9yrs 7m). This rather interesting discovery suggests that early nonverbal short-term memory is linked with the more complex, higher order mathematics (as in Mayberry & Do, 2003; Reuhkala, 2001) as opposed to early mathematics (Holmes & Adams, 2006; Holmes et al., 2008). Furthermore a recent study noted that different components of working memory had different relationships with different mathematical skills (Simmons et al., 2012) and whilst this study is not strictly an analysis of curricular mathematics skills it does bear some relation to the present chapter as it takes skills such as magnitude judgement and number writing into account. Magnitude judgement is a skill that is attributable to number sense and Simmons et al found that performance on nonverbal short-term memory skills uniquely predicts variance in scores on magnitude judgement tasks.

Krajewski and Schneider (2009a) present a model of early mathematical development that appears to encompass the ideas of “number sense” with subsequent developments that scroll through a lower level of numerical understanding inclusive of basic numerical skills, such as quantity discrimination, recitation of numerical words, exact number-word sequence (i.e. counting without understanding of quantity). A second level takes account of imprecise quantity linking (including concepts such as more than, lots, a bit), and precise quantity number linking, where the child is learning that a number (as a word) has a value associated and that this value can be concrete (as in two pencils on a desk) or abstract (two knocks on a door).

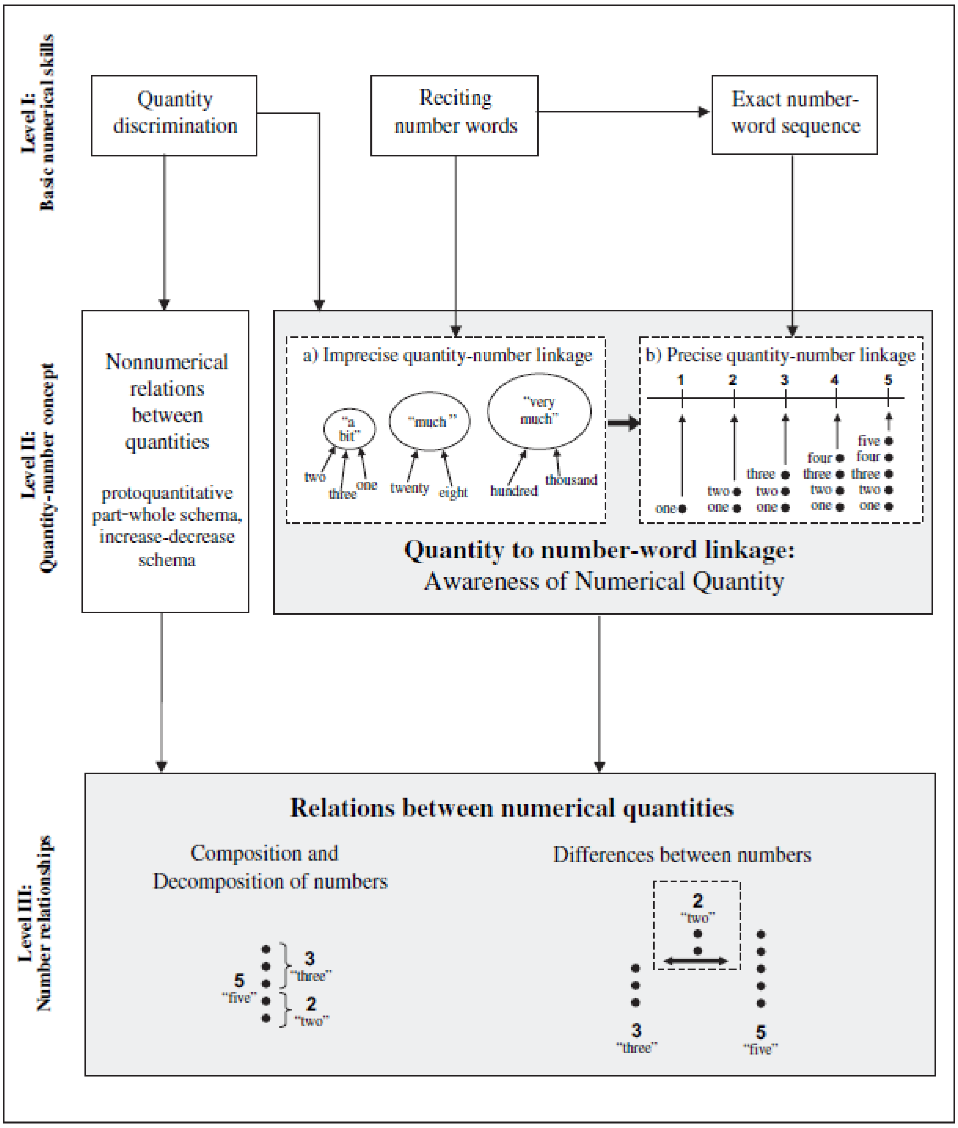

Figure 4. Krajewski and Schneider's model of early mathematical development (Krajewski & Schneider, 2009a)

In the early stages of the model Krajewski and Schneider discuss quantity discrimination which could also be interpreted as being related to the Approximate Number System. The Approximate Number System (ANS) is described as "a primitive mental system of nonverbal representations that suppose and intuitive sense of number in human adults, children and infants…" (page 1, Mazzocco, Feigenson, & Halberda, 2011). The numerical approximations are typically imprecise in infants, and they improve gradually over time. A number of studies have implicated the ANS as being predictive of latter school mathematical performance (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2013; Mazzocco et al., 2011). Research into the influence of working memory in the ANS and non-symbolic arithmetic processing are typically conducted with pre-schoolers, and are thought to further understanding into the cognitive architecture underpinning mathematical achievement (such as Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013; Xenidou-Dervou, van Lieshout, et al., 2013). Interesting findings are emerging, for example Xenidou-Dervou and colleagues (2013) conducted a dual-task study with pre-schoolers to examine active interference in all working memory domains during a non-symbolic approximate addition task. The results of this study indicated that there was a significant impairment in approximate addition performance under the central executive interference condition. The researchers discuss the theoretical ramifications of some results arising from this paper. It had previously been considered that approximate addition fell under the same theoretical umbrella as the ANS due to results found by Gilmore et al (Gilmore, Attridge, & Inglis, 2011)however Xenidou-Dervou et al suggest that non-symbolic addition tasks call upon different underlying cognitive processes, and in particular they believe that working memory underpins this type of task.

The third level in the Krajewski and Schneider model could be argued as the level whereby calculations become necessary, as it discusses the differences between numbers and also the ideas that numbers can be decomposed and composed from other numbers. These levels of mathematics development are indicated in Fig.5. (c.f. Krajewski & Schneider, 2009a). In terms of the Number Strand it might be expected that the role of nonverbal short-term memory would include the identification (but not use of) of mathematical stimuli such as symbols (+ and -), with a further role perhaps for estimating the numerical significance of groups or arrays of items in terms of numerical equivalence, and concepts like more and less. Furthermore there is the expectation that verbal short-term memory may have an active role in the Number Strand based on this evidence, so whilst this thesis has not used tasks measuring phonological awareness it does take account of verbal short-term memory and verbal working memory. It is understood from previous research that some tasks of phonological awareness seem to share variance with tasks of short-term and working memory (such as Word Recall and Listening Recall (see Leather & Henry, 1994)). Therefore it is appropriate to take account of some pertinent findings regarding verbal measures of the working memory model from the research looking at children presenting with mathematical disabilities (MD). Typically MD children will be identified as having weak or incomplete networks of number facts in long-term memory (Geary et al., 1991) which will understandably impair their competency in mathematics. There is also an awareness that children identified as having MD show deficits in verbal short-term memory and are less likely to use direct memory retrieval to solve mathematics questions (Bull & Johnston, 1997). Given this, V-STM is thought to be a key mechanism in the acquisition of number facts during early childhood, as V-STM is supposed to help shape complete networks of learned number facts into storage in long-term memory. In addition to supporting the laying down of number fact networks, verbal short-term memory is also thought to facilitate the retrieval of learned number facts from long term memory when children begin to use direct retrieval solution strategies (Dehaene & Cohen, 1997).

In summary there is considerable evidence that working memory is important in overall mathematical abilities, and in unpacking the mathematics task into the core strands that have been identified by, and taught according to the school curriculum, this study may be able to identify WM as important in the fundamental Number Strand building blocks that will inevitably support later mathematics learning and competency.

6.4 Aims and Research Questions

The key research questions arising in this chapter are:

1. The scant previous curriculum-based literature leads to a tentative hypothesis that the working memory model will be predictive of overall performance on the Number Strand at each time point (with reference to overall mathematics curriculum see Gathercole & Pickering, 2000a; and in "strand specific" terms Holmes & Adams, 2006).
2. There may be an influence upon the Number Strand of both nonverbal short-term memory (NV-STM) and phonological processes (V-STM). Supporting evidence for this hypothesis arises from functional visuospatial deficits identified in NV-STM that impact upon basic mathematical competency (Gathercole & Pickering, 2000b; McLean & Hitch, 1999), poorer curricular performance in this aspect of mathematics in typically developing

children (Holmes & Adams, 2006) and nonverbal (Simmons et al., 2012) and phonological (Krajewski & Schneider, 2009a, 2009b) contributions to symbolic mathematics, numerical equivalence and quantity representation.

3. It is also proposed that in the early stages of "Number" there will be a specific role for the Central Executive-CWM as the number words, Arabic symbols, the language of mathematics are identified, learned, committed to, and retrieved from LTM (Zuber et al., 2009).

4. Considering the working memory model and prediction of future performance on the Number strand, it is anticipated that early working memory measured at school entry will be predictive of subsequent Number Strand performance when measured two years later (see Holmes and Adams (2006) for similar interpretations regarding curricula based mathematics).

6.5 Methodology

The methodology for this section is the same as that detailed in Chapter 4. However, in order to be able to deconstruct the mathematical strands the data has been separated into the "strands". At each age grouping there were six questions that were specified by the Mathematics 5-7 Curriculum Links Sheet as belonging to the Number Strand. Raw scores from the Number Strand questions were then summed and converted to z scores to standardise the data about the mean scores of the sample. This method will allow us to draw meaningful comparisons with the other four strands in the overall discussion. Table 11 indicates some examples of Number Strand questions, all questions are shown in Appendix E.

Table 11. Sample items from the Number Strand

Maths 5 (n=6)	This shows the numbers you can press on a telephone. There are three numbers missing. Write in all the missing numbers.
	Look at the box at the top. There are **six** dots in it. Fins another box which has the same number of dots. Put a tick inside the box.
Maths 6 (n=6)	What number is 10 more than 7? Write your answer in the box.
	In the box write any number that is greater than 3 but less than 12.
Maths 7 (n=6)	Here are four numbers. Find the right name for each number.
	Which of these numbers is nearest to two hundred and fifty? Put a tick on it.

6.6 Results

6.6.1 Descriptive Statistics

Table 12 provides the descriptive data for performance on the Number Strand for each year grouping. At each age grouping Number was represented by 6 questions with mean scores ranging between 3.06 and 4.21 items correct.

Table 12. Descriptive Statistics for scores on Number Strand (n=70)

	Number			
	Questions (n)	Mean	Std. Dev.	Range (Min-Max)
Reception	6	4.21	1.56	6(0-6)
Year One	6	3.18	1.17	5(0-5)
Year Two	6	3.06	.123	5(0-5)

6.6.2 Correlational Analyses

The correlations between scores on the Number Strand are represented in Table 13, and Table 14 depicts the one-tailed zero order correlations between the raw scores of the working memory measures and the z scored Number Strand over three years. Age will be statistically controlled for in the subsequent regression analyses.

Table 13. Correlations between the Number Strand at each year grouping (n=70)

		1	2	3
1	**Number Strand Reception**	-		
2	**Number Strand Year One**	.44*	-	
3	**Number Strand Year Two**	.34*	.48*	-

* p=<.01

6.6.2.1 *Reception*

In the Reception year typically strong within-construct correlations were evident between the working memory domain specific pairings (ranging between *rs=.38 to .51, p=<.01)* as shown in Table 5. Across the construct domains it is noted that Nonword Recall (verbal STM) correlated with Listening Recall (verbal WM) but not with any of the other WM tasks. Marginally weaker effect sizes are apparent between V-STM and NV-STM indicating the likely functional independence of those WM domains. At this age grouping all WM measures correlated with the Number Strand except for Nonword Recall.

6.6.2.2 Year One

In this year grouping Table 6 identifies that all WM measures are correlating with each other across all theoretical WM domains. Additionally they are correlating with the mathematics outcome, Number Strand. Again typically strong effect sizes are shown between the domain specific pairs. Both measures of V-STM are showing strong correlations with Number at this age grouping *(rs = .56 and .61).*

6.6.2.3 Year Two

In the final testing phase it is apparent that Mazes Memory is no longer showing a correlation with the Number Strand *(r=.16 p=>.05),* nor with its domain specific NV-STM partner Block Recall *(r=.14, p=>.05).* Arguments regarding domain specificity versus task presentation modality have already arisen in the previous chapter (Pickering, 2001; Pickering et al., 2001; Pickering et al., 1998) and it is believed that this discrepancy between the domain specific correlations of the NV-STM measures is demonstrative of the age range whereby the presentation modality becomes an important factor.

Table 14. Zero order correlations between working memory measures, Performance Measures, and the Number Strand (one tailed)

		Number	
	Reception	**Year One**	**Year Two**
Working Memory			
Word Recall	.24*	.56**	.38**
Nonword Recall	.14	.61**	.28**
Listening Recall	.26*	.48**	.53**
Odd One Out	.44*	.51**	.41**
Mazes Memory	.36*	.46**	.16
Block Recall	.34*	.40**	.41**
Performance Measures			
Block Design	-	.23*	.36**
Object Assembly	-	.42*	.35**

* p<.05, ** p<.01
n.b. Performance Measures were not assessed at Reception.

6.6.3 Regression Analyses

6.6.3.1 Reception

The first model accounted for 14% of the unique variance in performance on the Number Strand at Reception age *(R^2 Δ=.14, p=<.05)* following elimination of any age related variance from the model. The standardised beta value for age in months was significant in Step 1, but this was no longer identified as a significant independent predictor variable by the standardised beta values (β) after including working memory measures to the model. Of the working memory measures, Odd One Out was noted as a significant independent predictor variable *(β=.25, t=2.03, p=<.05).* ANOVA Step 1: *(F (1, 68) =16.22, p=<.001)* and Step 2: *(F (7, 62) =4.44, p=<.001)* shows that this model provides a significantly better than chance prediction of performance on Number Strand. IQ (as indexed by Performance Measures) was not included at this age grouping.

6.6.3.2 Year One

In Model 2 (Table 15) working memory significantly accounted for 25% of the variance in performance on the Number Strand in Year One *(R^2 Δ=.25, p=<.001)* after removing age related and Performance Measures variance from the model (age in months significantly predicting 19% of the unique variance in scoring on the Number Strand). Examination of the β values inform us that Nonword Recall is emergent as a significant independent predictor *(β=.36, p=<.05).*

6.6.3.3 Year Two

In the final model (Year Two) overall the working memory model significantly contributed 17% of the unique variance in performance on this strand, Performance Measures 13%, and age only accounted for 8% of the variance. Via the standardised beta values it can be noted that the V-WM measure Listening Recall contributed significant independent variance to Number even after all the other variables had been taken into consideration. None of the other βs were significant.

Table 15. Hierarchical regression models predicting performance on the Number Strand, controlling for age and Performance Measures.

	Model 1: Reception Regressor: Number 5							Model 2: Year One Regressor: Number 6							Model 3: Year Two Regressor: Number7						
Predictor Variables : Order of inclusion	R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β	R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β	R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β
Step 1	.44	.18	.19	16.22**	.06	.03	.23	.20	.19	.19	16.76**	.04	.03	.16	.08	.07	.08	5.90*	.02	.04	.07
Age in Months																					
Step 2	-	-	-	-				.27	.23	.07	3.05*				.21	.17	.13	5.24***			
Performance Measures																					
Object Assembly					-	-	-					-.03	.03	-.13					.05	.03	.18
Block Design					-	-	-					.03	.03	.16					.01	.02	.06
Working Memory	.58	.26	.14	2.20*				.52	.44	.25	5.17**				.38	.28	.17	2.76*			
Word Recall					-.00	.04	-.00					.01	.06	.04					.07	.06	.16
Nonword Recall					.01	.04	.03					.13	.06	.36*					-.03	.07	-.05
Listening Recall					.02	.05	.04					.06	.05	.13					.14	.06	.34*
Odd One Out					.07	.04	.25*					.02	.04	.08					-.00	.04	-.02
Mazes Memory					.02	.05	.06					.04	.04	.13					.01	.03	.03
Block Recall					.05	.03	.21					.02	.03	.06					.03	.03	.12
ANOVA	ANOVA [f(7,62)=4.44, p=<.0001]							ANOVA [f(7,62)=7.12, p=<.001]							ANOVA [f(7,62)=4.04, p=<.001]						

Number 5 * p=<.05, **p=<.001
Number 6 * p=<.05, **p=<.001
Number 7 * p=<.05, **p=<.005

6.6.4 Working Memory at Reception Predicting Number Strand at Year Two

A regression analysis (Model 4) was undertaken to consider the predictive worth of working memory when WM is assessed at Reception age, upon Number Strand performance 2 years subsequent. The results from this analysis are highlighted in Table 16. Working memory is not contributing any significant variance in Number Strand performance.

Table 16. Hierarchical regression model predicting Number Strand at Year 2 with WM measures measured 2 years previous, controlling for age and Performance Measures.

Predictor Variables : Order of inclusion	Model 4 Regressor: Number Strand (Y2) R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β
Step 1	.08	.07	.08	5.90*	.03	.04	.10
Age in Months at Year 2 (Y2)							
Step 2	.21	.17	.13	5.24**			
Performance Measures							
Object Assembly Y2					.03	.03	.16
Block Design Y2					.03	.02	.24
Step 3	.29	.18	.08	1.12			
Working Memory (Reception)							
Word Recall					.05	.04	.15
Nonword Recall					-.01	.04	-.02
Listening Recall					-.02	.05	-.06
Odd One Out					.04	.04	.16
Mazes Memory					-.07	.06	-.19
Block Recall					.05	.03	.21
ANOVA	ANOVA [f(9,60)=1.66, p=<.01]						

*p=<.05, **p=.001

The standardised beta values (β) show that from the working memory measures, none were approaching significance as independent predictor variables. Similarly, neither of the Performance Measures was statistically significant independent predictors of performance on this Strand but the combined Performance Measures (as a rudimentary index of IQ) contributed 13% of the unique variance in performance on the Number Strand. Age at Y2 also reached statistical significance in terms of the unique variance provided.

6.7 Discussion

The intention of this chapter was to unpack the influence of working memory upon a specific identifiable educational domain or “Strand”, namely the Number Strand. This Strand has been defined by the NNS (DfEE, 1999) as providing the building blocks for children to understand number, counting, and the properties of number and number sequences, including negative numbers. The aims of this study were fourfold and each hypothesis will be discussed.

6.7.1 Is working memory model predictive of overall performance on the Number Strand cross sectionally?

The first aim of the longitudinal study was to examine whether the working memory model can be seen as a useful cross-sectional cognitive predictor of the Number Strand.

Taken as a whole conceptual model, cross-sectionally the working memory model is shown to significantly predict between 14% and 25% of the unique variance in scoring on the Number Strand of the Mathematics 5-7 test. To ensure that the unique variance in scores on this Strand is being captured the data analysis method utilised hierarchical regression models that accounted for age related variance and also Performance Measures that provided a rudimentary index of IQ. The percentages detailed here are broadly comparable with the data from the preceding chapter (Chapter 5) where WM predicts between 17% and 36% of the unique variance in mathematics overall scores in a cross-sectional analysis. The finding supports and extends earlier literature that suggests that there is a significant relationship between WM and National Curriculum performance (Gathercole & Pickering, 2000b; Holmes & Adams, 2006; Jarvis & Gathercole, 2003) by taking a single identifiable element of curriculum based mathematics and examining it in statistical detail using a cross-sectional design and regression techniques.

Passolunghi and Lanfranchi (2012) identified that both working memory and processing speed predict early numerical competences in kindergarteners, and that working memory only had an indirect effect upon mathematical achievement (longitudinally) and that this relationship was mediated by numerical competence. Subsequent research by Toll and Van Luit (2013) also identified those children with working memory deficits experienced difficulties in early numeracy skills, and that their developmental path shows a delayed profile when compared with typically developing working memory skills. In several other studies we have been able to ascertain that to an extent “mathematics predicts mathematics” (De Smedt, Verschaffel, et al., 2009; Geary et al., 1999; Jordan, Glutting, & Ramineni, 2010; Jordan, Glutting, Ramineni, et al., 2010; Jordan et al., 2013; Jordan, Kaplan, et al., 2010; Locuniak & Jordan, 2008), whereas the Passolunghi and Lanfranchi study and the Toll et al research show that there are working memory precursors to early numeracy as well as early mathematics (albeit indirectly). In both these papers the children were of a comparable age to the participants in the present study, and while the Number Strand does not directly compare with early numeracy tasks, it does go some way to supporting the current literature and expanding it with a measure directly related to the UK curriculum. Another finding identifies that the age of the child was significantly predictive of Number Strand competency in each year grouping within the cohort analyses, however in the final testing phase the amount of unique variance contributed to Number by the age of the child was hugely decreased. This may indicate that the crucial period for acquiring Number strand type information is within the first two years of formal schooling.

6.7.2 Are nonverbal short-term and verbal short-term memory specifically related to Number Strand performance?

There was a tentative proposition that NV-STM and V-STM would both have some predictive value upon the performance on the Number Strand subset of questions. The NV-STM hypothesis was largely based on Holmes and Adams curriculum study (2006) and the Simmons et al study identifying NVSTM as a predictor of magnitude judgement (Simmons et al., 2012). A correlational relationship was certainly evident between NV-STM and the Number Strand; however this association was all but eliminated in the regression analyses, which illustrates that NV-STM was not a significant independent predictor to the Number Strand performance cross sectionally. The implication arising from this is that NV-STM influence over mathematics would be in providing a supporting mechanism for the rest of the WM system to be able to perform their individual roles more successfully, such as direct retrieval from LTM and immediate storage and processing functions. This is assumed to be in part symptomatic of the NV-STM system being heavily supported by external tokens, be that by way of pictorial representations on the test paper, or items used in the task, such as blocks, coins and regular shapes (Ball, 1992).

With regard to influence of verbal short-term memory, during the middle testing phase (Year 1) a measure of V-STM was singled out as the most significant independent predictor variable. It is thought that this may be indicative of a period in Number Strand development where the children are being taught to, or are spontaneously beginning to utilise verbal codes to a greater effect (Dehaene & Cohen, 1997; Houdé, 1997). This idea is substantiated with evidence from others as this is around the age range that has previously been specified as the lower end of the scale in a developmental time window where children also begin to develop their phonological rehearsal skills (Gathercole et al., 1994; Gathercole & Hitch, 1993). This is around the time/age range that children would begin to start using subvocal rehearsal skill spontaneously. As children grow older, their speech rate increases, and so too does their rate of subvocal rehearsal. Faster rates of rehearsal permits more material to be maintained in the phonological store and continuously rehearsed without temporal decay, and so may lead to greater memory spans. Additionally, further evidence pertaining to the findings of this study argue that the increased involvement of verbal short-term memory in the older children's performance may reflect the mastery of symbolic-linguistic arithmetic (Holmes & Adams, 2006; Houdé, 1997) or mature solution strategies (such as direct retrieval) that rely on a verbal code (e.g., Dehaene & Cohen, 1997), and indeed it is found that there is a stronger relationship between Number Strand and non-numerical phonological measures of STM. This indicates that children are utilising phonological codes as opposed to visual and spatial codes to comprehend numerical information more effectively. A secondary explanation to be aware of may be related to the test administration. In each mathematics test all questions had to be read out loud by the experimenter. As such one may expect that in order to comprehend the questions a proportion of the variance being provided by verbal short-term memory is allocated to supporting transcoding from verbal codes to appropriate numerical codes (Zuber et al., 2009), however it would be expected that the weight of this would be largely similar across the three cross-sectional years and this is not the case here.

6.7.3 Working Memory (CE-CWM) and the Number Strand

The study hypothesised that CE-CWM would be an important predictor variable (Bull et al., 1999; Bull & Scerif, 2001) and it was found to be significant in the early Number Strand competencies at two of the three time points in the regression analyses. The Number Strand items generally do not require the pupil to process or calculate mathematics; the strand is more concerned with the representations of number, the principles of understanding number and counting, and the properties of number and number sequences. This study proposes that as this educational strand requires the commitment of a considerable volume of numerical information to long term memory storage, that the CE-CWM system is facilitating this process. While CE-CWM is typically associated with the processing element of mathematics, it is also understood that CE-CWM (and possibly the episodic buffer (Baddeley, 2000)) play a considerable role in facilitating the commitment of information to a long term store (Kaufmann, 2002).

Regarding analysis of Performance Measures as rudimentary measures of IQ included in the year-on-year study it can be statistically specified that while the IQ measures have a small predictive role neither of those measures realises potential as a significant independent predictor, whereas WM measures do at all three time points, even after the variance contributed by IQ measures is removed. In respect of CE and IQ sharing variance and perhaps being indicative of a more unitary construct (Fry & Hale, 2000) it is believed that this provides some more evidence for CE-CWM as a construct separable from IQ.

6.7.4 Can Number Strand performance be predicted by WM longitudinally?

Lastly it was hypothesised that the working memory model would be predictive of subsequent Number Strand performance when WM was measured at school-entry and the Number Strand was measured two years later. This was not found to be true. The longitudinal performance on the Number Strand was better predicted by the general Performance Measures which were assessed as a general index of non-verbal IQ. Krajewski and Schneider (2009a, 2009b) argued that early nonverbal short-term memory was linked with the more complex, higher order mathematics as opposed to early math, therefore it would be expected that early competency in NV-STM might be predictive of later math. In terms of the effect of NV-STM in context with the Number Strand, and no evidence is found that NV-STM is influencing performance on this strand longitudinally. However it is acknowledged that the Number Strand is not indexing complex, higher order mathematics, and is more akin to "early math". Appendix B shows the questions from the Number strand and out of a total eighteen questions only two also tap into the Calculation strand, and as such this study does not really address the third level of mathematical development proposed by Krajewski and Schneider (2009a). Finally a child is typically expected to arrive at primary school with some basic knowledge of counting and it should be noted that a distinct weakness in the present study was a failure to take account of early counting ability (even merely counting recitation) before commencing the study so the influence of prior counting knowledge upon this strand cannot be ruled out.

6.8 Summary

1. In summary, it can be concluded that while the WM model does have a significant influence in predicting performance on Number over a three year period of early mathematics development, its subsequent predictive value is diminished by both age and general non-verbal IQ. Our suggestion is that this is a result of increased automaticity of the representations of number and associated information over a period of time by means of support from central executive and commitment to LTM.

Chapter Seven

7 Working Memory and the Calculation Strand

> *Calculation is the mathematical strand concerned with understanding number operations and relationships, and rapid mental recall of number facts. It also deals with mental calculation, including strategies for deriving new facts from known facts. This chapter examines the relationship between measures of working memory and the Calculation Strand.*

7.1 The Calculation Strand

Within the UK curricular framework (DfEE, 1999; DfEE & QCA, 1999b) teachers are required to teach children "Calculation". Calculation is the mathematical "strand" which, by definition involves the understanding of number operations and relationships between numbers and operations, and rapid mental recall of number facts. The strand also takes into account written and mental calculation, including strategies for deriving new facts from known facts. By Year 1 for example children should be learning the operations of addition and subtraction and the related vocabulary, and they should recognise that addition can be done in any order. They would also be expected to know key arithmetic facts by heart, typically addition/subtraction facts up to and including 5 (e.g., 2 + 2 = 4, 4 – 3 = 1) and also begin memorising number bonds to 10. This clearly is not an exhaustive account of the kind of calculations that children from Reception to Year Two would be expected to perform, but it is an indication of the very basic levels of understanding necessary at the mid age grouping from this study. Chapter 3 provides more detail about the breadth of learning within the Calculation strand.

7.1.1 Working Memory and Calculation

As discussed in the preceding chapters it is generally understood that both the verbal and nonverbal aspects of short-term memory (V-STM – NV-STM) and CE-CWM impact upon general mathematical performance (Adams & Hitch, 1997, 1998; Alloway & Alloway, 2010; Andersson, 2007; Bull & Espy, 2006; Bull et al., 1999; Holmes & Adams, 2006; Holmes et al., 2008). Moreover it has also been reported that verbal and nonverbal short-term memory could have differential influences upon separable mathematical tasks (Bull et al., 2008; Noël et al., 2004; Passolunghi & Cornoldi, 2008) depending upon both the age of the cohort studied and the type of mathematics task being carried out.

Simple and complex calculations are among the mathematical areas that have been the most closely studied under the working memory umbrella. Simple calculation is generally considered to be single digit addition and subtraction (Adams & Hitch, 1997; Hecht, 2002) while complex calculations would cover calculations which will involve a succession of stages where parts of the sum would be executed and stored in memory until the subsequent step is completed (Adams & Hitch, 1997). Given that calculating even a simple mathematical equation will involve either fact retrieval from long-term memory (i.e. learned number bonds) or storage/processing combination, it can be inferred that working memory is likely to play a role in facilitating this process.

Tronsky (2005) comments that all components of working memory are involved in even simple arithmetical calculation, however other psychologists have attempted to identify if any singular working memory component is more important in the specific domain of “calculation” (Berg, 2008; Fuchs et al., 2010; Swanson, 2006a). Swanson (2006) found that the best WM predictor component of calculation was the visuospatial sketchpad element (NV-STM), substantiating research by Gathercole and Pickering (2000a, 2000b) that found that visuospatial abilities, as well as measures of central executive processing (CE), were associated with curricular attainment levels for children 6–7 years of age. Berg (2008) concurs that NV-STM is important in calculation and also finds that verbal working memory, a more specific sub-component of the central executive is a factor influencing performance on calculation tasks. Simmons and colleagues (Simmons et al., 2012) have shown that working memory has different relationships with different aspects of mathematics, and in this study they present data that shows the CE to be a significant predictor of single-digit addition, the most simple of calculations. They also measured single digit multiplication in Year 3 children and found evidence that the relationship between working memory and accuracy on this task was non-significant. This is in opposition to a number of other studies (Holmes & Adams, 2006; Passolunghi, Mammarella, & Altoe, 2008; Swanson & Beebe-Frankenberger, 2004) and Simmons et al argue that this may be due to the nature of the mathematics tasks used in the other studies. Those other studies used tests of more complex mathematical performance, covering a wider range of mathematical skills such as multi-digit calculations and problem solving; therefore it seems that it is important to attempt to separate out the individual mathematical skills to get a clearer picture of the influence of working memory upon mathematics.

A study by McKenzie, Bull and Gray (2003) examined the importance of phonological and visuospatial codes in simple arithmetic performance in two age groups (6 -7 and 8-9 yrs. of age). In this dual task study the younger cohort were largely unaffected by phonological interference but were shown to have impaired performance on the arithmetic tasks when faced with visuospatial interference. They also showed that the older children had impaired performance under both interference conditions. The inferences drawn are that the younger children rely more heavily on visual and spatial codes to facilitate simple addition, but the older children are likely to be using a combination of codes to expedite the calculation processes. It may be appropriate to argue that this could be resultant of the teaching and learning of new or more appropriate strategies; or it could be indicative of the fact that the phonological loop is less well developed with the younger children (Gathercole et al., 1994; Gathercole & Baddeley, 1993a).

In an examination of children's simple addition skills, Geary and Burlingham-Dubree (1989) reported that individual differences in the precision of using strategies for solving addition problems (counting on, counting all, or finger counting, or direct memory retrieval) was associated with spatial ability. In this study the child was measured on a strategy-choice variable which identified the accuracy of using both types of strategy. The study notes that the differences in strategy-choice variable were not related to language ability but were significantly correlated with Geometric Design and Mazes tasks from the Wechsler Preschool and Primary Scale of Intelligence (Wechsler, 1974). Given that these tasks required duplication of both simple and complex patterns and spatial scanning, Geary and Burlingham-Dubree suggested that children depended upon spatial information and strategies such as combining concrete representations to add and solve arithmetic problems, and that this may contribute to the relationship between performance on numerical and spatial ability measures.

Some more recent work has examined the involvement of working memory in children's exact and approximate mental addition (Caviola et al., 2012) with some interesting results regarding the differentiation between the effects of WM on approximate and exact calculations in two presentation modalities (horizontal and vertical addition tasks). They found that in slightly older children (aged between 8 and 9 years), approximate calculation is more demanding of WM resources than exact calculation in children, and that no differences were found between exact and approximate calculations for either the percentage of correct responses or the mean correct latency, indicating that both approximate and exact calculations present the same degree of difficulty to children. They also note that the performance on the addition task is affected by the presentation format of the question, with horizontally presented additions being more impaired than vertical ones when faced with a verbal interference task, and that the vertical addition task items were more disrupted by visuospatial interference. In another dual task study with children aged between 10 and 12 years old Imbo and Vandierendonck (2007b) concluded that executive resources are implicated in arithmetic performance (large number single addition, e.g. 7 + 9) but developmentally the effect of central executive decreased over time on these types of mathematics sums. This gives rise to the assumption that the reason for this is that as children progress through the school system they merely become more skilled at simple calculations and increasingly experienced at quickly accessing the most appropriate solution strategies.

7.1.2 Working Memory and Calculation Longitudinally

Longitudinally, Noël, Seron and Trovarelli (2004) find a link between verbal short-term memory and verbal complex memory with calculation performance in French children in Year 1 (aged 5-6) showing that both the phonological loop tasks and verbal based central executive tasks measured at the beginning of Year 1 were significantly predictive of addition performance when addition was measured four months later. This study makes the inference those children with poor phonological skills resort to using immature counting strategies instead of using memory processes to complete simple additions. However Passolunghi, Vercelloni and Schadee (2007) partially disagree based upon their findings that it was only the central executive part of working memory that demonstrated significant predictive power over mathematics in children with an average age of 6 years and 4 months, when mathematics was measured six months after the WM measures were taken.

In a Belgian longitudinal study De Smedt and colleagues also identified a clear predictive role for the central executive (2009) in both first- and second-grade[4] mathematics achievement. The De Smedt et al study measured mathematics achievement by way of a Flemish curriculum based test; and the test covered aspects of mathematics other than merely calculation. De Smedt et al found that CE was a predictor variable both four months and twelve months subsequent to the initial working memory tasks being completed. They also noted that there were age-related differences with regard to the contribution of the slave systems to mathematics performance; the visuospatial sketchpad was a unique predictor of first grade, but not second grade mathematics achievement, whereas the phonological loop emerged as a unique predictor of second grade, but not first-grade, mathematics achievement.

Geary et al (1991) also considered working memory and addition in young children over a period of time (first and second graders), and also included children with mathematical disabilities. Geary and colleagues suggest that a principal factor contributing to an early learning problem in mathematics is a difficulty in the retrieval of basic information from long-term memory. Across times of measurement the typically developing children showed an increased reliance on memory retrieval, with fewer retrieval errors, and a decreased reliance on strategies such as finger counting or counting aloud to solve addition problems. This indicates that visuospatial and phonological reliance is waning whilst central executive skills are supporting speedy arithmetical

[4] In both the USA and Belgium the age of first grade students is 6-7 years, and second grade is aged 7-8 years.

fact (in this case, addition facts) retrieval from LTM. From each of these studies working memory is longitudinally implicated as key in facilitating calculation abilities in children, and specifically applying to the age range between five and seven years old.

A potential confound in some of the studies discussed (for example De Smedt, Janssen, et al., 2009; Geary et al., 1991; Imbo & Vandierendonck, 2007a; Passolunghi et al., 2007) is the use of numerical working memory tasks such as digit span, counting span, backwards digit span etcetera. It has been discussed in a previous chapter (Chapter 5) that the use of numerically grounded working memory tasks may have a strong influence on the resulting data given the task similarity with the majority of mathematics type tasks. This study has provided evidence that in two of the three years of our data collection there was a significant difference in the relationship between numerical working memory tasks upon mathematics overall and as such it was considered appropriate to exclude numerically based tasks from the analyses in this thesis.

While it is clear throughout this chapter, and indeed the thesis, that there is extensive agreement in the literature that WM holds an important role in calculation ability and development, there is also still considerable dispute as to the exact nature of the role of the sub-components of WM. As this thesis highlights, very few researchers have attempted to look at WM in conjunction with the teaching policies of the National Numeracy Strategy and the more recently updated Primary National Strategy, as such there is a distinct lack of literature that directly precedes this work. A key influential study conducted by Holmes and Adams (2006) endeavoured to examine the implications of working memory on mathematical development and the UK mathematics curriculum. The youngest cohort in the Holmes and Adams work were in Year 3 in primary school, which meant that they were aged around seven to eight years old.

Holmes and Adams used regression models to show that phonological loop scores did not account for any unique variance in “mental arithmetic” scores (akin to calculation in the present study) above and beyond that accounted for by age and the other two WM constructs that they measured. Models to determine the predictive value of NV-STM indicated that visuospatial ability accounted for a very small, but still significant amount of variance in mental arithmetic scores, just 1%. Finally, the CE models illustrated that from the WM measures, central executive scores accounted for the greatest amount of unique variance in both overall mathematics scores and the mental arithmetic subset. After the variance contributed by age and visuospatial and phonological scores is accounted for then CE further accounted for 22% of variance in mental arithmetic scores. While the findings from the Holmes and Adams paper (Holmes & Adams, 2006) are clearly the most relevant to the present study, it is felt that direct inferences that the results will be comparable cannot yet be made, as there is the expectation that the way WM and calculation interact may change and develop from the very early school years over time, and the Holmes and Adams cohort were slightly older than those children taking part in the present study.

7.2 Aims and Research Questions

The key research questions arising in this chapter are:

1. The pertinence of the Holmes and Adams (2006) study leads us to hypothesise that the working memory model as a whole will be predictive of the Calculation Strand when considered cross-sectionally.
2. It is also proposed that CE-CWM will be the strongest predictor of performance on calculation tasks (De Smedt, Janssen, et al., 2009; Holmes & Adams, 2006; Passolunghi et al., 2007).
3. Thirdly, working memory at school entry will be predictive of performance on the Calculation Strand when the maths assessment is undertaken two years following school entry. Some researchers have demonstrated that working memory has a longitudinal predictive value on mathematics in general over varying periods of time (De Smedt, Janssen, et al., 2009; Geary et al., 1991; Passolunghi et al., 2007)

7.3 Methodology

The complete methodology was described in Chapter 4 and summarised in Chapter 5. To briefly recap the variables measured are indicated in the table below.

Table 17. Working memory and mathematics measures assessed.

Cognitive Domain	Task 1	Task 2
Verbal Short Term Memory (V-STM)	Word Recall	Nonword Recall
Central executive (CE-CWM)	Listening Recall (Verbal)	Odd One Out (Nonverbal)
Nonverbal Short Term Memory (NVSTM)	Mazes Memory (Visual Static)	Block Recall (Spatial Dynamic)
Performance Measures (Index of Nonverbal IQ)	Block Design	Object Assembly
Mathematics	Calculation Strand	

Examples of the questions from the Calculation Strand are shown in Table 18 and covered fully in Appendix D.

Table 18. Sample items from the Calculation Strand

Maths 5 (n=3)	Look at the first box. This shows that Tola had four balloons. Then she blew up three more. Draw all Tola's balloons in the next box.
	Which domino has seven dots altogether? Put a tick under it.
Maths 6 (n=13)	Double each of the numbers in and write your answers in the boxes.
	How many pairs of socks are there?
Maths 7 (n=8)	The question says "what must be added to 8 to make 17?"
	Here are two number machines. The first one adds three to any number that you put in. What does the second machine do?

7.4 Results

7.4.1 Descriptive Statistics

Table 19 shows that there is an inequality in the number of items measuring the Calculation Strand and this chapter later analyses performance on the Calculation Strand using hierarchical regression. In order to meaningfully analyse the data the raw scores on the Calculation Strand were summed then standardised to produce a z score. This was undertaken due to the unequal quantity of questions in the maths test that pertained to each individual strand in order that the results for each strand could be compared.

Table 19. Descriptive Statistics for scores on Calculation Strand (n=70)

	Calculation			
	Questions (n)	Mean	Std. Dev.	Range (Min-Max)
Reception	3	1.47	1.01	3(0-3)
Year One	13	6.54	2.91	10(1-11)
Year Two	8	3.03	1.74	6(0-6)

7.4.2 Correlational Analyses

Reported in Tables 5, 6 and 7 (Chapter 5) are the one-tailed zero order correlations between each of the working memory measures and the partial correlations after controlling for age related variance. It was deemed unnecessary to repeat these data in each subsequent chapter; therefore Table 20 references the correlations between the Calculation Strand at each age range, and Table 21 the working memory measures, the performance measures and Calculation Strand for each year group. Age is to be statistically controlled for in the subsequent regression analyses.

Table 20. Correlations between the Calculation Strand at each year grouping (n=70)

		1	2	3
1	**Calculation Strand Reception**	-		
2	**Calculation Strand Year One**	.67*	-	
3	**Calculation Strand Year Two**	.59*	.73*	-

* p=<.01

The within construct interactions between the WM measures were described in full in section 6.6.2 of the previous chapter.

These analyses identified that verbal short-term memory did not correlate with calculation in the first year of testing (Reception), but at both subsequent time points all working memory constructs correlated significantly with the Calculation Strand at their relative time points, with rs ranging between .30, p=<.01 for Listening Recall and Calculation at the Reception time point and .68, p=<.001 for Listening Recall and Calculation at the Year 1 time point.

Table 21. Zero order correlations between working memory measures, performance measures and the Calculation Strand at each age range, one tailed (n=70)

	Calculation		
	Reception	**Year One**	**Year Two**
Working Memory			
Word Recall	.17	.59*	.29*
Nonword Recall	.01	.64*	.36*
Listening Recall	.30*	.68*	.59*
Odd One Out	.48*	.63*	.63*
Mazes Memory	.43*	.46*	.35*
Block Recall	.34*	.47*	.37*
Performance Measures			
Block Design	-	.53*	.49*
Object Assembly	-	.36*	.28*

*p=<.01
n.b. Performance Measures were not assessed at Reception.

7.4.3 Regression Analyses

Regression analyses appraised the amount of unique variance in performance on the Calculation Strand that WM accounted for cross-sectionally (Table 22). In order to examine the regression results further, each section will be broken down by the hypotheses stated earlier in the chapter.

7.4.4 Working memory and the Calculation Strand

At the Reception time point the hierarchical regression analyses demonstrated that the WM model as a whole accounted for 20% of the unique variance in Calculation over and above that of the variance derived from age *(R^2 Δ=.20, p=<.001)* (Table 18). It is recognised that the quantity of questions pertaining to Calculation at the Reception time point is very small, and therefore the results from this should be interpreted with caution. At the second time point in Year One working memory as a whole is significantly accounting for 32% of the unique variance in scores on Calculation, on top of the variance pertaining to both age and Performance Measures. Similarly at the third time point it is found that WM is contributing significantly to the calculation scores with the R^2 change statistic showing at *.26, p=<.001*. These data show that WM is a significant cross-sectional predictor of performance on the Calculation Strand even when age and Performance Measures have been statistically controlled for.

7.4.5 A specific role for CE-CWM when predicting performance on Calculation

There is a slight divergence as to which of the standardised beta values achieves significance at each of the three time points. In Reception a statistically significant β is identified for Odd One Out *(β=.30, p=<.05).* At Year One the standardised beta values indicate that the unique contributors to the R^2 change figure are both Nonword Recall (V-STM) and listening recall (V-CWM) *(β = .32, p=.<05 and β = .34, p=<.001)* respectively. At Year Two three working memory measures are significant independent contributors (Listening Recall β *= .32, p=<.005,* Odd One Out β *= .33, p=<.005* and Mazes Memory β *= .20, p=<.005*).

It is noticeable that overall, CE-CWM is the strongest predictor of calculation cross-sectionally over a three year period, but it seems that the passive elements of working memory (V-STM and NV-STM) do also have a small but significant role to play. It appears in Year One and Two that both NV-STM and V-STM interchangeably support calculation performance, but not so in Reception year.

Table 22. Hierarchical regression models predicting performance cross-sectionally on the Calculation Strand, controlling for age.

	Model 1: Reception Regressor: Calculation							Model 2: Year One Regressor: Calculation							Model 3: Year Two Regressor: Calculation						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1	.16	.14	.16	12.59***	.06	.03	.22	.26	.25	.26	23.52**	.03	.02	.10	.12	.11	.12	9.14**	.03	.03	.10
Age in Months																					
Step 2	-	-	-	-	-	-	-	.38	.35	.13	6.71*				.31	.28	.19	9.03***			
Performance Measures																					
Object Assembly					-	-	-					.01	.02	.04					-.01	.02	-.03
Block Design					-	-	-					.03	.02	.15					.03	.01	.21
Step 3	.36	.28	.20	3.22**				.70	.66	.32	10.72**				.57	.50	.26	6.02***			
Working Memory																					
Word Recall					-.02	.04	-.08					-.01	.04	.-02					-.04	.04	-.13
Nonword Recall					-.03	.04	-.10					.10	.04	.32**					.06	.05	.14
Listening Recall					.05	.05	.12					.13	.03	.35**					.11	.04	.32**
Odd One Out					.08	.03	.30*					.04	.03	.17					.07	.03	.33**
Mazes Memory					.07	.05	.18					.00	.03	.02					.05	.02	.20*
Block Recall					.02	.03	.08					.01	.02	.03					-.02	.02	-.10
ANOVA	ANOVA [f(7,62)=4.91, p=<.0001]							ANOVA [f(9,60)=15.70, p=<.0001]							ANOVA [f(9,60)=8.76, p=<.0001]						

Calculation 5 * p=<.02, **p=<.005, ***p=<.001; Performance measures not assessed
Calculation 6 * p=<.005, **P=<.001
Calculation 7 * p=<.005, **p=<.005, ***p=<.001

7.4.6 Predicting performance on the Calculation Strand when the maths assessment is undertaken two years following school entry

A further regression analysis was calculated to examine the predictive value of working memory when assessed at Reception age upon mathematics performance 2 years subsequent (see Table 23). Comparable with the cross-sectional analyses in the tables above, the consistent predictive factor emerging from the WM variables is central executive, in particular Odd One Out (NV-CE). This is after any unique variance accounted for by Age at Y2 and Performance Measures at Y2 has been removed from the statistical equation. Both age and Performance Measures are statistically significant predictor variables; however the variance that is attributable to those variables is smaller than that predicted by WM.

Table 23. Hierarchical regression models predicting Calculation performance at Year 2 with WM measures measured 2 years previous, controlling for age and Performance Measures.

	Model 4 Regressor: Calculation (Y2)						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1	.12	.11	.12	9.14**	.02	.03	.09
Age in Months at Y2							
Step 2	.31	.28	.19	9.03***			
Performance Measures							
Object Assembly Y2					-.01	.02	-.03
Block Design Y2					.04	.02	.31**
Step 3	.57	.46	.23	4.80***			
Working Memory (R)							
Word Recall					.07	.04	.22
Nonword Recall					-.07	.04	-.21
Listening Recall					..05	.04	.12
Odd One Out					.09	.03	.32**
Mazes Memory					.03	.02	.07
Block Recall					.02	.03	.07
ANOVA	ANOVA [f(9,60)=7.58, p=<.0001]						

*p=<.05, **p=<.01, ***p=<.001

The R^2 change statistic *(R^2 Δ=.23)* is demonstrating that even after all of the other key cognitive factors have been taken into consideration, WM measured at Reception age is contributing circa 23% of the unique variance in scoring on the calculation part of the mathematics test when assessed 2 years later. The β values indicate one significant independent predictor variable, namely Odd One Out.

7.5 Discussion

This thesis chapter attempts to better clarify the relationship between working memory and the mathematics Calculation Strand. Based upon the preceding literature the hypothesis was that working memory as a whole would be predictive of the specific academic domain of Calculation cross-sectionally. Additionally, it was proposed that central executive would be the strongest predictor of performance on calculation tasks. This chapter also aimed to discover if Calculation could be predicted longitudinally.

Results of the current study provide further evidence for the role of working memory and related cognitive processes in Calculation in typically developing children (Adams & Hitch, 1997; Holmes & Adams, 2006; Swanson, 2006a). It also builds upon previous findings that WM predicts wider aspects of National Curriculum attainment (e.g. Gathercole & Pickering, 2000a, 2000b; Holmes & Adams, 2006) and that WM assessments may be useful as early predictors of scholastic attainment.

7.5.1 Working Memory Predicting Calculation

This study shows working memory model to be a significant predictor of calculation performance in the cross-sectional analyses; the results suggested four important findings. Firstly, CE-CWM emerged as a consistent significant independent contributor of Calculation at each year grouping. Second, age related variance and Performance Measures did not eliminate the contribution of working memory to Calculation. Third, in the latter two time points, individual short-term memory components (i.e., verbal STM and nonverbal STM) interchangeably contributed unique variance to Calculation, as denoted by beta values in the regression analyses. Fourthly, the full regression model highlighted that age remained a significant contributor to Calculation in the presence of significant contributions from all other variables. The results are consistent with the hypothesis, both supporting and extending the findings of Holmes and Adams (2006) with the inclusion of a younger cohort and the addition of supplementary working memory variables. It is also evident from the cross-sectional analyses that the most significant independent predictor variables are those two that are measuring central executive which is also consistent with some of the previous findings (Bull et al., 1999; Bull & Scerif, 2001). Furthermore this also strongly concurs with Holmes and Adams (2006), where they show that the central executive predicts 22% of the unique variance in mental arithmetic, and this figure is comparable with statistical data in the present study. In trying to separate out the Calculation Strand this chapter endeavoured to tease apart the involvement of the working memory model in calculation specifically, and as such the findings of Simmons et al (2012) are relevant. The results of this study do not support Simmons et al, however it does accept their assertion that, despite best efforts to narrow the Calculation Strand to a single mathematical competency it is apparent that this Strand actually includes tasks that are beyond the scope of simple arithmetic as in Simmons et al. This chapter suggests that working memory continues to predict a substantial proportion of the variance in other mathematical skills (over and above simple and multi-digit calculations), not only as those skills are higher order mathematics, and more complex, but also because there is a necessary learning aspect to all mathematics in school, and this requires constant commitment of information to long-term memory stores.

Through regression analyses it is found that the influence of verbal short-term memory upon the Calculation Strand is most apparent during Year One, and to a lesser degree in Year Two. These data appear to be forming a similar pattern to the data from the Number Strand (chapter 6). In the previous chapter the likelihood that this is indicative of a period in time where the phonological loop is in development is discussed (Gathercole et al., 1994; Gathercole & Baddeley, 1993a). Moreover the visuospatial sketchpad appears to be important in explaining individual differences in Year Two Calculation performance. Holmes and Adams (2006) have suggested that the visuospatial sketchpad (NV-STM) provides a workspace for representing abstract mathematical knowledge in a concrete form and our finding shows similarities to other studies (Holmes et al., 2008; McKenzie et al., 2003) but in our cohort the children were at least one year younger than those in either study mentioned. Rasmussen and Bisanz (2005) indicated that pre-schoolers frequently used concrete representations for doing arithmetic, such as fingers and objects, and that the use of these concrete representations required visuospatial working memory. Siegler (1997) also reports that a child will typically default to immature strategies like finger counting sooner than attempt to retrieve an answer directly from memory, and that this is not manifest in a strict developmental sequence, and it is felt that this is apparent in the present study. This is thought to be so as the Calculation Strand tasks require more active processing and the child is faced with more difficult calculations, then s/he will revert to back up strategies using visual encoding that involve nonverbal short-term memory resources, thus utilising the abstract-to-concrete mental workspace.

7.5.2 A robust relationship between CE-CWM and Calculation Strand

A number of studies have shown a relationship between CE-CWM and calculation and mental arithmetic type tasks in children (Berg, 2008; Bull et al., 1999; Geary et al., 1991; Holmes & Adams, 2006; Passolunghi & Siegel, 2001; St Clair-Thompson, Stevens, Hunt, & Bolder, 2010; St Clair-Thompson, 2011). As previously mentioned, in the year-on-year analyses these data are comparable with the Holmes and Adams (2006) findings, identifying that working memory is accounting for between 20% and 32% of the unique variance in performance on the Calculation Strand after all other measured variables had been taken into consideration with the greatest independent predictor variable overall being central executive. Looking at each time point in turn, it can be seen that during the Reception year CE-CWM is a significant independent predictor variable and at the second time point V-STM is also predicting the Calculation Strand alongside CE-CWM. However at the final time point nonverbal short-term memory is recognised as also influencing the outcome measure of Calculation in tandem with central executive. This finding appears to substantiate the McKenzie et al (2003) findings to some degree, insofar as it can be seen that while CE-CWM remains a strong predictor overall, the relationships between the other two aspects of working memory appear to be shifting developmentally. However, Imbo and Vandierendonck (2007b) put forth the notion that CE-CWM involvement in Calculation tasks decreases over time. The present findings do not substantiate this, and it is believed that the influence of CE on Calculation may remain at a consistent level given our three year cross-sectional findings. The correlational analyses demonstrate that as CE-CWM span develops over the first three years of formal schooling, the relationship between CE-CWM and Calculation remains stable. It is accepted that the generalizability of these findings can only apply to the age range that this study has assessed, but can be further substantiated with the work of Holmes and Adams (2006), as the younger children in their cohort were a year older than those in this present study and the mathematics tasks that they used were broadly similar in content, whereas Imbo and Vandierendonck (2007b) have reported findings from an older age range of children. It is of course possible to argue that the older age groups have stronger representations of concrete number facts and may be less reliant upon CE-CWM, but it is certainly apparent that between the ages of 5 and 7 years old, children consistently utilise CE-CWM processes to facilitate performance on Calculation tasks.

7.5.3 Predicting performance on the Calculation Strand at age 7 using WM measures taken shortly after school entry

Based upon some of the prior evidence (De Smedt, Janssen, et al., 2009; Noël et al., 2004; Passolunghi et al., 2007) it was hypothesised that the early measures of the working memory model would be predictive of subsequent Calculation Strand performance. The longitudinal correlational design shows that early working memory is a reasonably strong predictor of later calculation performance, accounting for 23% of the unique variance in performance on this Strand. This figure is not diminished by the presence of variance from age, nor from the Performance Measures. The strongest independent predictor variable was Odd One Out, which is a measure of nonverbal working memory. That this measure of working memory (OOO) is strongly correlated with Digit Recall at Reception might indicate that children are utilising a numerical type code to be able to perform the Odd One Out task. Perhaps converting the shapes in the boxes into numerical codes, or using a counting technique to fulfil the needs of the task. This is purely speculative but the high degree of variance shared by these two variables is of some interest and could be explored in greater details. Furthermore, agreement is noted with other longitudinal research such as de Smedt et al (2009) (where calculation was assessed a year later). It is thought that as this thesis is examining a separable strand of mathematics, this study extends the evidence found for a concurrent relationship between mathematics in general and working memory which has been identified in previous cross-sectional research (e.g. Adams & Hitch, 1998; Gathercole, Pickering, Knight, et al., 2004; Holmes & Adams, 2006). The findings of Passolunghi et al (2007) and De Smedt et al (2009) can be extended with evidence that WM is not only predictive of calculation cross-sectionally, and over a short period of time (between 4 months to 1 year respectively), but also it enduringly predicts scoring on the Calculation Strand when calculation is measured two years following the initial working memory measures.

7.6 Summary

1. This study focused on a discrete set of cognitive processes related to performance (a rudimentary index of IQ) and working memory in order to decipher which measures could most accurately predict achievement on the Calculation Strand of the Mathematics 5-7 tests. From this it has been ascertained that CE-CWM is the most stable and statistically significant independent predictor of Calculation both at each year of testing and longitudinally.
2. This chapter and the one previous also demonstrate that there may be an emerging pattern of evidence to suggest that there is a differential effect of measures of short-term working memory on separable mathematical tasks cross sectionally.
3. Longitudinally the most significant independent predictor variable from the working memory measures is Odd One Out (NV-CWM). The fact that working memory measured at the beginning of school entry is predicting 23% of the unique variance in scores on the National Numeracy Strategy/Primary National Strategy Calculation Strand suggests that early working memory assessments may prove to be useful as diagnostic tools for highlighting potential problems in this key aspect of curricular mathematics.

Chapter Eight

8 Working Memory and the Problem Solving Strand

The focus of this chapter turns to the effects of working memory upon mathematical problem solving as defined under the auspices of the UK curriculum. The topics that are covered by the Problem Solving Strand include making decisions about which method of calculation is appropriate to use; completing word based mathematics problems, and reasoning about numbers and making general statements about them.

8.1 The Problem with the "Problem Solving" Strand

When one thinks of problem solving in a mathematical sense, arithmetic word problems are typically considered to be the most sizeable part of the issue (e.g., Peter had eight sweets. He gave two sweets to his friend. How many sweets did Peter have left?). The typical explanation of such word problems is that they are linguistically presented, single- or multi-step problems requiring arithmetic solutions, but evidently that type of task is not all encompassing in the realms of mathematical problem solving.

The Problem Solving Strand in the UK curricular structure is primarily concerned with factors such as making decisions: deciding which operation and method of calculation to use (mental, mental with jottings, pencil and paper, calculator), reasoning about numbers or shapes and making general statements about them, solving problems involving numbers in context: 'real life', money, and measures, and not just arithmetical word based multistage problems (see Appendix E for learning outcomes for the Problem Solving Strand).

Given this dichotomy between the curricular strand of Problem Solving and the perception of mathematical problem solving as a measureable task, an attempt must be made to link Problem Solving (the curricular strand) with the previous psychological literature. It is probably most straightforward to focus on those more tangible aspects of problem solving in a mathematical sense, which have been measured before. Therefore the focus is on arithmetical word problem solving.

8.1.1 Working memory and current thinking on Problem Solving as arithmetical word based problems

A significant body of work has concentrated on cognitive predictors of arithmetical word based problems solving tasks (Andersson, 2007; Fuchs et al., 2006; Fuchs et al., 2010; Hecht, 2002; Kail & Hall, 1999; Lee, Ng, Ng, & Lim, 2004; Meyer et al., 2010; Passolunghi & Mammarella, 2010; Swanson, 2006b; Swanson, 2011; Swanson & Beebe-Frankenberger, 2004; Swanson et al., 2008), but there has been considerably less research on the relationship between the UK curriculum, Problem Solving as a taught concept and the contribution of working memory (Holmes & Adams, 2006). Therefore the study is drawn towards research into working memory and arithmetic word problem solving.

Early research indicated that individuals with higher levels of working memory capacity have a tendency to perform better on learning tasks because they have more cognitive resources available to them (Daneman & Carpenter, 1980). Furthermore the effect of working memory on mathematical problem solving has been documented in a number of studies (Andersson, 2007; Swanson, 2006b; Swanson & Beebe-Frankenberger, 2004; Swanson et al., 2008; Swanson & Sachse-Lee, 2001; Zheng et al., 2011). Much of this research indicates that central executive functions seem to be among the key predictors of children's performance in solving mathematical word problems as well as in written mathematical calculation (see Bull et al., 1999; Bull & Scerif, 2001; Gathercole & Pickering, 2000b; Gathercole, Pickering, Knight, et al., 2004; Lee et al., 2004; Swanson & Beebe-Frankenberger, 2004). More recently Kyttälä and colleagues (Kyttälä, Aunio, Lepola, & Hautamäki, 2013) have specified that nonverbal CWM was having a direct effect upon performance in children's arithmetic word problem solving. Relevant to this Chapter, Kyttälä et al also use the Odd One Out task and these data show nonverbal working memory made a significant contribution to word problem solving above and beyond the role of general intelligence and age.

Turning briefly to deficits in arithmetical problem solving tasks, Passolunghi and Siegel (2001) suggested that when IQ was matched in peer groups, poor problem solvers were still performing at a lower level on working memory tasks than their IQ matched peers with better problem solving skills. This finding implies that working memory has an influence upon arithmetical word problem solving that is above that of IQ.

8.1.2 Calculation as a mediator between working memory and Problem Solving

Research consistently finds that basic calculation (addition and subtraction with sums less than 20) covaries with mathematics achievement (Durand, Hulme, Larkin, & Snowling, 2005; Geary & Brown, 1991; Hecht, Torgesen, Wagner, & Rashotte, 2001) and Problem Solving is a measurable aspect of mathematics whereby it is theoretically appropriate to consider calculation as being an influential factor upon achievement in the Problem Solving Strand. In a cross -sectional study with a slightly older cohort than those children in the present study Andersson (2007) showed that three measures associated with central executive and one measure associated with verbal short-term memory contributed unique variance to mathematical problem solving when the influence of reading, age and IQ were controlled for *(r^2 =.39, p=<.05).* Andersson conducted a further regression model predicting arithmetic problem solving that included calculation as a predictor variable and he found that the amount of variance predicted increased to 63% *(p=<.05)* suggesting that working memory and calculation both have a strong role in children's problem solving performance .

Where Andersson (2007) suggests that the inclusion of calculation to the regression model strengthens the ability to predict arithmetical problem solving this study takes the view that calculation may actually mediate the relationship between working memory and problem solving. Chapter 7 has already ascertained that working memory significantly influences performance on the Calculation Strand (between 20% and 32% of the variance in scores on Calculation, after controlling for age and IQ). It is expected that there will be a significant relationship between working memory and problem solving, but since a mathematical operation is frequently required to find a solution to a *typical* arithmetical problem, it might be reasonably assumed that calculation ability will mediate the relationship between working memory and Problem Solving.

8.2 Aims and Research Questions

There are three key aims regarding the relationship between Problem Solving in terms of a curricular strand, and working memory;

1. In line with previous chapters, the exploration of the relationship between working memory and Problem Solving is to be assessed using regression analyses. It is proposed that working memory as a whole will be predictive of Problem Solving.
2. It is also hypothesised that the CE-CWM component will be an independent predictor of Problem Solving in the cross-sectional analyses (similar to Kyttälä et al., 2013).
3. The study wanted to further explore the unique contribution of working memory to children's mathematical problem solving when mediated by Calculation (identified by Andersson, 2007). Based upon Andersson (2007) it is anticipated that Calculation will mediate Problem Solving. This analysis is exploratory, and largely based on the premise that there are conflicting opinions as to the significance of the effects of working memory upon mathematical problem solving (Andersson, 2007; Kail & Hall, 1999; Swanson, 2006b; Swanson & Beebe-Frankenberger, 2004; Swanson, Cooney, & Brock, 1993). To undertake these analyses modern statistical mediation methods were used (Hayes & Preacher, 2011).

The resulting data and analysis will be divided into two sections, primary regression analyses and secondary mediation modelling.

8.3 Regression Methodology

The methodology for this section is the same as is detailed in Chapter 4. Raw scores from the Mathematics 5-7 Curriculum Links Sheets determining which questions were attributed to Problem Solving (NFER-Nelson, 2001) were summed and converted to z scores to standardise the data about the mean scores of the sample. This was a necessary step as the quantities of questions at each year group, relating to each strand were not always equal.

Appendix G shows all of the questions that are attributed to the Problem Solving Strand and examples of the type of question found in the Problem Solving Strand are also shown in Table 24.

Table 24. Sample items from the Problem Solving Strand

Maths 5 (n=5)	The man and the lady want to buy a cupboard. The cupboard has to be shorter than the man but taller than the lady. Which cupboard is the only one they can buy?
	In this question you will have to choose more than one coin. You want to buy a pencil. It costs seven pence. Which coins make seven pence? [coins of various denominations in front of the child]
Maths 6 (n=13)	The question says, sweets cost 4 pence each. Katie buys 2 sweets. How much does she spend?
	Ten children were asked whether they like tomatoes. The number of children who like tomatoes is shown inside the circle. How many children do not like tomatoes? [6 stick men in a circle and 4 stick men positioned outside the circle.]
Maths 7 (n=16)	Find two odd numbers that add up to 8. Remember both numbers must be odd.
	A group of children were asked which fruit they liked best. This [graph] shows how many children chose each fruit. Which fruit do most children like? Write your answer on line A. How many more children like bananas than plums? Write your answer on line B

8.4 Results

8.4.1 Descriptive Statistics

The number of Problem Solving items in Reception was low in comparison with those in both following years. Descriptive data is reported in Table 25. The means and range in Year One indicated that there may be an issue with the items in the test at this age grouping.

Table 25. Descriptive Statistics for scores on Problem Solving Strand (n=70)

		Problem Solving		
	Questions (n)	**Mean**	**Std. Dev.**	**Range (Min-Max)**
Reception	5	2.63	1.05	5(0-5)
Year One	13	1.84	.97	3(0-3)
Year Two	16	5.50	2.09	8(1-9)

8.4.2 Correlations

Table 26 identifies the correlations between the Problem Solving Strand at each age group. It can be noted here that the correlations between this strand at each age group are lower than might be anticipated.

Table 26. Correlations between the Problem Solving Strand at each year grouping (n=70)

		1	2	3
1	**Problem Solving Strand Reception**	-		
2	**Problem Solving Strand Year One**	.25*	-	
3	**Problem Solving Strand Year Two**	.46**	.30**	-

* p=<.01, ** p=<.05

As the next step in exploring the contribution of working memory to Problem Solving, correlations were calculated among all tasks used in the study. The results of these correlations are presented in Table 27. It is interesting to note that all variables, across each of the three time points yield significant correlations with Problem Solving apart from verbal short-term memory measures at time 1 as also found with Calculation (Chapter 7), and Block Design of the Performance Measures at time 2. An additional note of the correlations between Problem Solving and Calculation is made to facilitate the mediation modelling (Reception: *r=.56, p=<.01*; Year 1: *r=.39, p=<.01*; Year 2: *r=.69, p=<.01*) (for detailed discussion about working memory and Calculation refer to Chapter 7).

Table 27. Zero order correlations between working memory measures, Performance Measures, the Calculation and the Problem Solving Strand at each age range, one tailed (n=70)

	Problem Solving		
	Reception	**Year One**	**Year Two**
Working Memory			
Word Recall	.10	.33**	.30**
Nonword Recall	.02	.35**	.37**
Listening Recall	.29**	.26*	.44**
Odd One Out	.54**	.37**	.41**
Mazes Memory	.29**	.30**	.32**
Block Recall	.43**	.24*	.24*
Performance Measures			
Block Design	-	.03	.31**
Object Assembly	-	.25*	.46**
Calculation	.56***	.39***	.69***

* p<.05, ** p<.01, p=<.001***
n.b. Performance Measures were not assessed at Reception.

8.4.3 Regression Analyses

8.4.3.1 Cross-sectional Analyses

To expedite a more detailed exploration of the relationships a succession of fixed-order hierarchical regression analyses followed (Table 28). These measure the amount of unique variance in the Problem Solving Strand predicted by each of the individual working memory measures after controlling for age related variance and any variance pertaining to the Performance Measures.

Model 1 (Reception year) indicates that the WM model accounted for 24% of the variance in Problem Solving Strand *(*R^2 *Δ=.24, p=<.001)* with a significant beta value identified for Odd One Out (NV-CWM), after eliminating any age related variance from the model *(β=.38, p=<.001).*

In both subsequent models for the two years following, the amount of variance accounted for by working memory has decreased (Model 2, R^2 *Δ=.11, ns*; Model 3, R^2 *Δ=.15, p=<.05)* and none of the individual working memory components were emergent as statistically significant independent contributors to that variance. An initial possibility was that it was quite likely that the slightly larger variance predicted by working memory in Model 1 (Reception year) could be due to the lack of Performance Measures at that age grouping, and as such some of the shared variance between WM and IQ is possibly being misdirected. However it can be seen that in Year Two working memory remains a significant predictor of Problem Solving, even after Performance Measures have been taken into statistical account.

Table 28. Hierarchical regression models predicting performance cross sectionally on the Problem Solving Strand, controlling for age.

	Model 1: Reception Regressor: Problem Solving							Model 2: Year One Regressor: Problem Solving							Model 3: Year Two Regressor: Problem Solving						
Predictor Variables : Order of inclusion	R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β	R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β	R^2	Adj. R	R^2 Δ	F Δ	B	SE B	β
Step 1	.23	.22	.23	20.38**	.10	.03	.39**	.06	.05	.06	4.33*	.01	.04	.04	.06	.05	.06	4.50*	.01	.03	.04
Age in Months																					
Step 2								.11	.07	.05	1.95				.24	.21	.18	7.86*			
Performance Measures																					
Object Assembly												-.05	.03	.24					.01	.02	.07
Block Design												.02	.03	.14					.04	.02	.27*
Step 3	.47	.41	.24	4.61**				.23	.11	.11	1.47				.39	.30	.15	2.39*			
Working Memory																					
Word Recall					-06	.04	-.19					.05	.06	.04					-.01	.04	-.03
Nonword Recall					-.02	.04	-.07					.01	.06	.18					.10	.06	.23
Listening Recall					.06	.05	.15					.04	.05	.02					.08	.05	.23
Odd One Out					.11	.03	.38**					.04	.04	.15					.02	.03	.08
Mazes Memory					-.03	.04	-.08					.01	.04	.15					.04	.03	.17
Block Recall					.04	.03	.18					.01	.03	.04					-.03	.02	-.15
ANOVA	ANOVA [f(7, 62)=7.79, p=<.0001]							ANOVA [f(9,60)=1.94, p=<.06]							ANOVA [f(9, 60)=4.24, p=<.001]						

Problem Solving R *p=<.01, **p=<.001; Performance measures not assessed
Problem Solving Y1 * p=<.05, **p=<.001
Problem Solving Y2 * p=<.05, **p=<.001

8.4.3.2 *Longitudinal Regression Analysis*

Model 4 (Table 29) is a hierarchical regression model to consider which aspects of working memory would impact upon later Problem Solving Strand performance. In this instance working memory does not significantly account for any of the unique variance in scores on Problem Solving. The only significant independent contributor to this performance on this strand was Block Design, of the Performance Measures), *(β=.29, p=<.05)* and the cumulative performance measure scores are significantly accounting for 18% of the unique variance in scores.

Table 29. Hierarchical regression model predicting Problem Solving performance at Year 2 with WM measures measured 2 years previously, controlling for age and Performance Measures.

	Model 4 Regressor: Problem Solving (Y2)						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1	.06	.05	.06	4.50*	.01	.04	.03
Age in Months at Y2							
Step 2	.24	.21	.18	7.86**			
Performance Measures							
Object Assembly Y2					.01	.03	.07
Block Design Y2					.04	.02	.29*
Step 3	.31	.21	.07	1.01			
Working Memory (R)							
Word Recall					.04	.04	.11
Nonword Recall					-.01	.04	-.04
Listening Recall					.02	.05	.04
Odd One Out					.05	.04	.19
Mazes Memory					.04	.05	.10
Block Recall					.00	.03	.01
ANOVA	ANOVA [f(9,69)=3.02, p=<.005]						

*p=<.05, **p=<.001

8.5 Discussion- Regression Analyses

8.5.1 Cross-sectional Data

As with the preceding chapters, the purpose of this section has been to better understand and specify any relationship between working memory and the mathematics Problem Solving Strand. Based upon the preceding literature it was hypothesised that working memory as a whole would be predictive of the specific academic domain of Problem Solving at each year grouping.

Additionally, it was anticipated that CE-CWM would be the best predictor of performance on problem solving tasks given the multi-step nature of problem solving tasks.

Age of participant was significantly related to Problem Solving at each age grouping, and Performance Measures had some predictive value at Year Two. The data only partially supports the first hypothesis, in that working memory is predicting a proportion of the unique variance in scores on the Problem Solving Strand at two of the three time points available. At these two time points (Reception and Year Two) age related variance and Performance Measures did not eliminate the contribution of working memory to Problem Solving. However the only year grouping where a significant individual independent working memory predictor was found was Reception (Odd One Out – CE-CWM), and this is the year grouping where Performance Measures were not assessed. The first assumption might be that as Performance Measures were not assessed, that some of the variance from the central executive measures is perhaps being misappropriated, as it is known that working memory and intelligence tend to share some variance. However, from the Year One data it can be noted that Performance Measures are not a significant predictor here either, but they are in the final year of testing. Unfortunately at this juncture I can only speculate that the impact of Performance Measures in Reception would not be a crucial predictive factor upon Problem Solving at Reception, although it is acknowledged that it is possible that some of the variance is being driven by intelligence/performance measures, and not necessarily wholly by CE-CWM.

At the central time point it is accepted that there appears to be a problem with the data. That the descriptive analysis showed an anomalous mean score and the range was correspondingly low, and coupled with the regression analyses showing no significant predictive value for working memory as a whole was a concern. This prompted a closer look at the questions in the Strand and the resulting data. The highest score obtained on this Strand in Year One was 3 out of 13. It was expected that an average score above 5 would be evident. Subsequent examination of the items in this Strand, at this age group revealed that a large number of the items pertained to monetary units and it was clear that many of the children performed badly on these specific items. This could indicate a problem with administering this test at a mid-point in the year. It suggests that the children may not have reached the point in the curriculum where money and coins had been taught.

8.5.2 Longitudinal Regression Analysis

The study also examined the working memory predictors of Problem Solving over time. The correlational analyses showed that Year Two Problem Solving was significantly related to all of the working memory skills assessed apart from Nonword Recall. However the longitudinal regression analysis identified that Performance Measures were the clearest predictor of Problem Solving at Year Two. Accounting for 18% of the unique variance of scoring on Problem Solving tasks after controlling for age related variance. Age also contributed a small but significant amount of variance in Problem Solving scores *(6%, p=<.05).* Therefore considering the predictive value or working memory measures taken at school entry upon Problem Solving at Year Two it is found that working memory is not a significant predictor but both age and Performance Measures appear to be important factors in explaining the variance in scores.

8.6 Mediation Modelling Methodology

The methodology for the testing period for the mediation modelling is the same as that described in Chapter 4 and previously in this chapter. With respect of the data analyses a decision was taken to only analyse the data from the final year of testing. There are several reasons for this decision. Firstly, in Reception year Performance Measures were not assessed, and for completeness in the mediation model it is necessary to account statistically for as much of the variance attributable to Problem Solving as possible. Secondly, in Reception year of testing there were only five questions asked as part of the Problem Solving strand, and on only one occasion was a calculation required. Thirdly, in Year One from the regression analyses a potential issue with the Problem Solving data was identified. There were 13 Problem Solving questions in total and the scoring on this strand ranged from 1-3, with a mean of *1.84 (sd =.97),* therefore this represents something of a floor effect in the raw scores. As such the following mediation model is presented as a snapshot of what is occurring between the predictor, the mediator and the outcome variables at Year Two.

8.6.1 Statistical Background

The primary goal of mediation analysis is to explain the mechanism that underlies an observed relationship between an independent variable and a dependent variable by including a third explanatory variable, known as a mediator variable. Modern approaches to statistical mediation analysis focus on estimation and inference about the indirect and direct effects of generally accepted cause *X* on presumed effect *Y* through proposed intervening variable *M*. The early causal steps approach that was described by Baron and Kenny (Baron & Kenny, 1986) is now reported to "leave a lot to be desired" (Hayes & Preacher, 2011). They argue this for two main reasons; firstly that it is "one of the lowest power methods available …" (p.4 Hayes & Preacher, 2011), and secondly that the approach does not emphasise the explicit quantification and inferential testing of the indirect effect. Hayes and Preacher (2011) discuss in detail the reasons why the Baron and Kenny method is now out of favour, and as such this chapter operates with in the more current statistical thinking about mediation analysis, utilising one of the Hayes and Preacher methods. To analyse this data PROCESS method was used (Hayes, 2013; Hayes & Preacher, 2011) , and for simplicity the PROCESS.spd add-in for SPSS 20 was deployed (Hayes, n.d) as this provides a clear graphical user interface and allowed the input of covariates and choose appropriate bootstrapping methods. A composite working memory score was derived by summing the scores across the working memory tests and obtaining the mean. This was necessary as one of the limitations of using PROCESS is that only one predictor variable is allowed to be input. A composite score of Performance Measures was also used as this model is not overly concerned with the relationship between Performance Measures and Problem Solving.

Also of benefit, this modelling technique allows for theoretically driven models, whereas in older mediation model methods predictor variables must conform to significance at each step before a mediator can be identified. That said causal inference can be strengthened if the researcher can argue or demonstrate that the variables have been modelled in the appropriate causal sequence.

There are two effects of *X* that are of primary interest in mediation analysis. Firstly if interest is the direct effect between the X and Y variables as depicted in Fig.5, but most central to mediation models is the indirect effect of *X*, on Y when M is statistically accounted for.

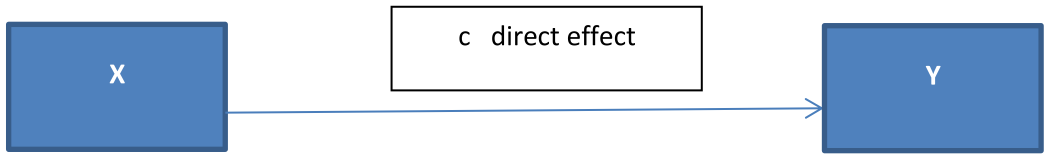

Figure 5. Simple diagram representing a direct effect (c) between an independent variable (X) and a dependent variable (Y)

This is quantified as the product of coefficients *a* and *b*. This product, *ab*, is interpreted as the amount by which two cases that differ by one unit on *X* are estimated to differ on *Y* as a result of the effect of *X* on *M* which in turn affects *Y*. This can be visually represented as in Fig.6.

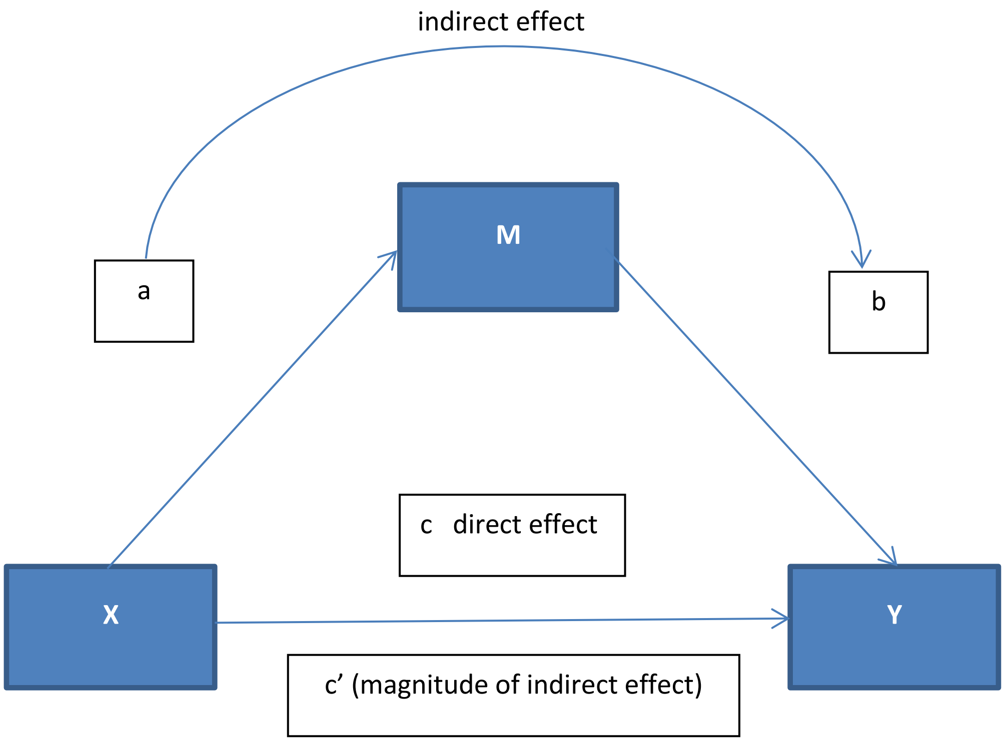

Figure 6. Diagram representing an indirect effect (ab) between an independent variable (X) and a dependent variable (Y), where c-c' represents the magnitude of the indirect effect.

The indirect effect of *X* serves as a quantitative tangible example of the mechanism through which *X* influences *Y*. But it is not the only path of influence from *X* to *Y*. *X* can also influence *Y* directly, independent of its indirect effect via *M*. C prime (*c'*) quantifies how much two cases who differ by one unit on *X* but who are equal on *M* are estimated to differ on *Y* and is represented as the magnitude of the indirect effect.

The terms indirect effect and mediating effect are often used interchangeably in the mediation literature, the preferred term for the remainder of the chapter is *indirect effect*. The full mediation model can be visually represented as in Fig. 6.

When performing the mediation analyses there were some differences in how the data was used compared to the regression analysis. Problem Solving and Calculation raw scores were used as opposed to standardized (z) scores, as for this section there is no attempt to meaningfully compare the resulting data with that of other chapters in this thesis. This section is much more focused upon the detailed analysis of the variables and their relationships with one another.

8.6.2 Bootstrapping

A brief note on bootstrapping samples indicates that:

> *"Bootstrapping is a technique from which the sampling distribution for a statistic is estimated by taking repeated samples from the dataset. In effect this treats the data as a population from which smaller samples are taken. The statistic of note is the beta coefficient (β), and this is calculated for each sample, from which the sampling distribution of the statistic is estimated. The standard error of the statistic is estimated as the standard deviation of the sampling distribution created from the bootstrap samples. From this confidence intervals and significance tests can be calculated", (adapted from Field, 2009 p.782).*

The resulting confidence intervals are identified as significant if the upper and lower levels confidence intervals do not pass through zero (denoted as *CL LL* and *CL UL*).

8.7 Mediation Modelling Results

The Hayes and Preacher PROCESS method (Hayes, 2013, n.d) allows the statistical modelling of the total (working memory composite) and indirect (via Calculation) effect upon Problem Solving. This model also allows for the inclusion of control variables, which in this case allowed for the control of variance from Age in Months and the Performance Measures composite upon both the M and Y variables.

The hypothesised association between Working Memory and Problem Solving mediated by Calculation is depicted in Fig. 7. Using the Preacher and Hayes macro for SPSS (Hayes, n.d.; 2008), 95% confidence intervals of the mediation effect were estimated using bootstrapping re-sampling (k = 5000) procedures (the bootstrap method replaces the inferior Sobel test; (Shrout & Bolger, 2002)).

8.7.1 Year Two Mediation Model

The relationship between working memory and Problem Solving was significantly mediated by Calculation. Working memory was a significant predictor of Calculation (*path a*, *b=.33, p=<.0001, t (67) =4.54, p=<.0001*), and of Problem Solving (*path c, b=.27, T (67)-2.82, p=<.0001*) and Calculation was also a significant predictor of Problem Solving scores (*path b*, *b=.71, t (67) = 4.98, p=<.0001*). As Figure 7 illustrates, the relationship between working memory and Problem Solving was decreased substantially when controlling for Calculation (*c' path b=.04, t (67) =.45, p=.66*). Bootstrapping estimates revealed that the model was statistically significant (*b= .23, CI LL .13, CL UL .38*).

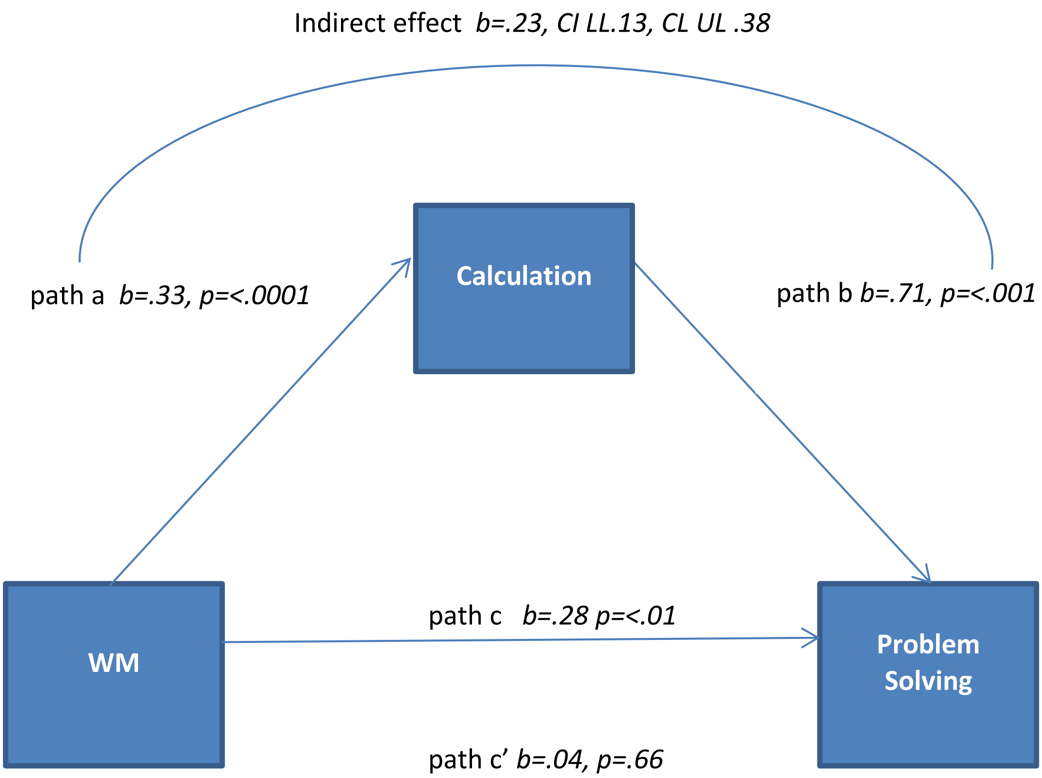

Figure 7. Year Two Mediation Model Results

8.8 Discussion - Mediation Modelling

In the case of the present research theoretical arguments that working memory is related to arithmetical problem solving have been discussed, and that working memory has a predictive value upon these type of mathematical tasks (Fuchs et al., 2006; Ostad, 1998; Swanson, 2006b; Swanson et al., 1993; Swanson et al., 2008; Swanson & Sachse-Lee, 2001; Zheng et al., 2011). Moreover Andersson (2007) has argued that calculation abilities may mediate the effect of working memory upon Problem Solving type tasks. Age and performance measures are reported to be predictors of mathematical performance in general, and working memory and performance measures can share some variance. So typically in regression models age and performance measures are controlled for in order that the proportion of variance in scores that is specifically attributable to working memory can be evaluated (Andersson, 2008). In order to examine this premise modern mediation modelling techniques were used (Hayes & Preacher, 2011; Preacher &

Hayes, 2004). Based on limited past research (Andersson, 2007) it was anticipated that there would be an indirect effect of Calculation upon the outcome variable Problem Solving. Andersson used regression analyses to investigate the contribution of working memory to mathematical word problems, a technique widely used in this thesis. However to try to elicit a more detailed account of the mediating influence of Calculation the Hayes and Preacher PROCESS method was used (Hayes, 2013, n.d). Andersson (2007) controlled for calculation ability as a step in the regression analyses and found that even after taking account of this variable some working memory variables (animal dual task and verbal fluency – both central executive, and digit span – verbal short-term memory) remained predictive of problem solving. Conversely the mediation analysis presented in this study show that Calculation is fully mediating the relationship between working memory and Problem Solving after taking the covariates of age in months and Performance Measures composite score into account. It is suggested that this finding is an interesting starting point for research in the area of Problem Solving. In the introduction the fact that problem solving transparently requires a calculation was discussed, and this has certainly been true in the previous research on arithmetical problem solving. However Problem Solving within the UK curriculum is not so clear cut. In the Mathematics 7 test there were a total of 16 questions attributable to the Problem Solving strand, yet only four of those questions actually required a calculation and the mediation model shows that performance on Calculation is fully mediating the relationship between working memory and Problem Solving.

8.9 Discussion

A number of difficulties were encountered when trying to tease apart the influence of working memory upon the Problem Solving Strand. It appears that the Problem Solving Strand as defined by the curriculum is a relatively abstract conceptualisation of problem solving, rather than a clear definition similar to that of Calculation in the previous chapter.

In teaching the Problem Solving Strand, typically the teacher would be concerned with helping pupils to understand decision making processes, assist in their ability to decide upon appropriate methods of calculation to use, such as using a calculator or pencil and paper, or mental arithmetic. Teachers would also be expected to facilitate reasoning about numbers or shapes and to help the student grasp real world examples of arithmetic problem solving such as monetary units etcetera. Some of these concepts are arguably difficult to measure, a child might choose the appropriate strategy, or make the correct reasoning judgements; however they may fail at the calculation stage of the problem. Whilst the Mathematics 5-7 test allowed the student to make notes or use the page to do working out, few children actually used this opportunity, and as such there was no way to measure if appropriate strategies/reasoning were employed. Bearing this information in mind Problem Solving tasks within the Mathematics 5-7 test (NFER-Nelson, 2001) were considered in more detail (see Appendix G for Problem Solving items from Mathematics 5-7). From this it is noted that there is the anticipated crossover with the Calculation Strand (similar to Andersson, 2007; Kail & Hall, 1999), but also it is also apparent that many of the questions have pictorial representations of a verbally presented problem, and as such there is an assumption that NV-STM would be less necessary as the visual representations scaffold those processes. In terms of the mediating influence of Calculation upon Problem Solving, Chapter 7 discusses in detail how working memory is a good predictor of performance on the Calculation Strand, accounting for around a quarter of the variance in scoring on the Strand and the mediation model bears this out. The mediation model shows that performance on Calculation is mediating the relationship between working memory and Problem Solving, and given these data it is thought that the influence of working memory over Problem Solving is effective via Calculation. Kyttälä et al (2013) have also been interested in the role that working memory has to play in arithmetic word problems, finding that the relationship between verbal working memory and arithmetic word problems was mediated by expressive vocabulary and listening comprehension. Their data also gave evidence that visuospatial working memory performance did predict word problem solving, which is somewhat inconsistent with the data presented in this thesis, however this thesis recognises the limitations of the data in this “strand”, in particular at the first two testing points. Another point to consider is that in the Kyttälä study they do not take account of the influence of calculation ability upon problem solving, and the model in this thesis indicates that the effect of working memory upon problem solving is almost reduced to nothing when calculation has been statistically modelled and accounted for.

Longitudinally, the results showed that measures of working memory do not predict problem solving independently of Performance Measures and age (Kail & Hall, 1999; Swanson et al., 1993). Similarly no conclusive evidence was found that any single working memory variable or component is important in this Strand longitudinally. There is a speculative notion that the visual and physical representations in the test are clouding the verbal aspects of the task, and perhaps masking any strong effects of working memory.

8.10 Summary

1. This first part of this chapter focused on a discrete set of cognitive processes related to performance (a rudimentary index of IQ) and working memory in order to best understand which measures could most accurately predict achievement on the Problem Solving Strand of the Mathematics 5-7 tests. From this it has been ascertained that CE-CWM is a significant independent predictor of Problem Solving at school entry and working memory in general is predicting performance at Year Two but not at the central time point.
2. It is of concern that there is a potential floor effect in the data at the central time point, despite the descriptive analyses showing that the data was of a reasonable quality. Following the regression analyses, the Year One data was evaluated in more detail, and it was noted that the highest score achieved on Problem Solving in Year One was 3 out of a possible 13, where would have expected a score above 5 (see Appendix G for questions on this strand).
3. In conclusion the regression analyses find that working memory is influencing the Problem Solving Strand at Reception and Year Two time point, but not at Year One due to the data issues reported. It is recognised that the lack of Performance Measures at

Reception, and the issues with the Problem Solving data at Year One are potentially problematic, and thus the results should be interpreted with a degree of caution.

4. In the second part of this chapter, when using modern statistical methods to analyse mediating effects, Calculation was found to be a significant mediating factor in the relationship between working memory and Problem Solving Working memory is understood to be a robust predictor of Calculation (chapter 7) and as such it seems that the influence of working memory upon Problem Solving is via the ability to perform Calculations, even when calculations are not vital in the questions set.
5. It is felt that there are some useful extensions to the previous literature in that widely recognised, standardised materials were used to measure both Problem Solving and working memory. In previous research the WM tasks used have varied (Andersson, 2007; Fuchs & Fuchs, 2002; Hecht, 2002; Ostad, 1998; Swanson & Sachse-Lee, 2001; Zheng et al., 2011) , as have the methods of measuring mathematics, the present study goes some way to rectifying this particular problem with the uniformity of the design.
6. However it is also felt that this chapter cannot adequately address the mediating role of Calculation in Problem Solving due to some inadequacies in the dataset, but this may be a useful starting point for further work in this specific area.

Chapter Nine

9 Working Memory and the Measures, Shape, and Space Strand

In this chapter the Measures, Shape and Space Strand is investigated in relation to associations with working memory. The Measures Shape, and Space strand includes choosing units and comprehension of scales such as time, rulers and temperature scales. It also covers choices between inches or centimetres, the properties of two and three dimensional shapes, position, distance, direction and movement.

9.1 The Measures, Shape and Space Strand

In this chapter the effects of working memory upon the Measures, Shape and Space Strand are assessed (DfEE, 1999; DfEE & QCA, 1999b). Within the UK curricular framework (DfEE, n.d.-c, n.d.-d) we find that the "strand" Measures, Shape and Space would expect pupils to arrive at Reception class equipped with the understanding of concepts such as "heavier", "lighter", "smaller". They would also be expected to be able to use basic everyday language related to time, order and sequence, as well as having an understanding of basic shapes such as circle and square. During the course of the academic year children should be developing the ability to use familiar objects to (re)create patterns and build models and be able to describe the position of objects (up, down, below). Throughout the first three academic years these basic proficiencies would be expected to extend to accommodate such concepts and skills as estimating and measuring using standard units (e.g. a metre rule or measuring jug), use vocabulary to represent time including days of the week, hours/half hours and followed by minutes and seconds. Pupils should further be able to visualise and name 2D shapes and 3D solids, and follow and give instructions involving position, direction and movement. Once again this is not an in-depth report of how children from Reception to Year Two would be expected to perform in this particular strand, but it provides a basic suggestion of the expectations upon the student as designated by the curriculum.

Given the particular focus of this mathematical strand upon shape, units of measurement, patterns and such like, coupled with the educational literature providing guidance for educators to facilitate learning in the MSS strand by the use of visual and tactile aids, particularly when teaching MD pupils (DfES, 2001) it seems logical and feasible to assume that there may be a specific role for visuospatial memory (NV-STM) with the type of mathematics that occurs in this strand. In the realm of general mathematics Rasmussen and Bisanz (2005) reported that NV-STM scores predicted unique variance in pre-schoolers' performance on nonverbal mathematics problems, suggesting that they may be adopting a mental model for arithmetic that requires the NV-STM resources. In other research NV-STM has been identified as a significant correlate of standardised mathematics attainment across a variety of age ranges within a school population (Gathercole & Pickering, 2000a; Jarvis & Gathercole, 2003; Mayberry & Do, 2003; Reuhkala, 2001).

As already discussed, to the best of the author's knowledge there are only two published papers that directly precede the idea that the separable curricular aspects of mathematics can be unpicked and measured alongside working memory (Holmes & Adams, 2006; Holmes et al., 2008). As a small part of each study the authors assess the Measures, Shape and Space portion of UK curriculum based mathematics. There are several other studies that have also discussed this mathematical strand, although unfortunately they have not published exact data or conclusions regarding the strand, but rather used a global score of mathematics in their results (see Gathercole et al., 2003; Gathercole & Pickering, 2000a; Gathercole & Pickering, 2000b). In attempting to illustrate the effects of age-related differences in the involvement of NV-STM in children's general mathematics McKenzie, Bull and Gray (McKenzie et al., 2003) can be considered, and it can be inferred that robust relationships between NV-STM and mathematics may be more apparent in younger children, and those relationships appear to diminish somewhat when looking at the mathematics attainment of older children. This is reflected in the 2008 study (Holmes et al., 2008) and in general these data are a good fit with previous influential working memory research with respect to broad-spectrum scholastic attainment (Gathercole & Alloway, 2004; Gathercole et al., 2003; Gathercole & Pickering, 2000b; Gathercole, Pickering, Knight, et al., 2004).

9.2 Working Memory and the Measures, Shape, and Space Strand

With regard to Measures, Shape and Space, in both the 2006 and 2008 studies (Holmes & Adams; Holmes et al.) the authors applied 15 questions in their mathematics test to assess this facet of mathematics with a slightly older cohort than the present thesis. Holmes and Adams (2006) evidenced strong correlations between a single measure of each CE and NV-STM with Measures, Shape and Space, and more detailed statistical analyses identified that CE best predicted performance on questions in the test pertaining to Measures, Shape, and Space, with NV-STM only accounting for a tiny portion of the variance in scores. In the 2008 study (Holmes et al., 2008) state that a composite VSSP score predicted greater variance in the younger children's total overall mathematics scores (Year 3 = 10%, $p<.05$; Year 5 = 3%, $p<.05$), which they interpret as evidence of an age related difference. However composite NV-STM did not significantly predict performance across the separable mathematical skills, including Measures, Shape and Space. Similarly the individual NV-STM tests showed no significant predictive value over this strand in particular. Despite Holmes studies (Holmes & Adams, 2006; Holmes et al., 2008) not finding a significant predictive relationship between NV-STM and Measures, Shape and Space, this aspect of curricular mathematics is still considered worthy of study as the children in the present research were considerably younger than those in the Holmes research.

Central executive(CE-CWM) has been identified as a predictor of mathematics (Bull et al., 1999; Geary et al., 1991) and in discussing the curricular structure of the Measures, Shape and Space strand this study can perhaps identify another more distinct mathematical area where central executive might be a contributory factor. It is known that this strand requires a considerable amount of information to be learned and committed to long term memory stores. Items such as shapes and their names, units of measurement, and concepts such as taller, shorter, heavier, are all examples of knowledge that this strand needs to have speedy access to in order for a child to perform with success (Baddeley, 1996a). Gathercole and Pickering (2000a) argue that executive processes are important in this type of processing as information of this nature is being processed, integrated with, and committed to long term memory and retrieved when needed. We also know that children who perform poorly on mathematics assessments will tend to have poorer central executive functioning, (Bull et al., 1999; Geary et al., 1999).

In summary, due to the marked lack of research into this specific curricular aspect of mathematics this chapter is largely exploratory. The main purpose of this section of the study was to inspect systematically the contributions of the three different components of the working memory model to the Measures, Shape and Space strand.

9.3 Aims and Research Questions

1. The primary aim of this section was to assess the contributions of the three different components of WM to the Measures, Shape and Space Strand within the UK Curriculum. The limited previous curricular research (Holmes & Adams, 2006; Holmes et al., 2008) and general WM/scholastic attainment research (Gathercole & Pickering, 2000a, 2000b) leads us to hypothesise that both CE-CWM and NV-STM will both show an influence over scoring on the Measures, Shape and Space facet of the Mathematics 5-7 tests (NFER-Nelson, 2001) both cross sectionally and longitudinally.
2. A secondary intention of the study was to examine any changes in the pattern of cognitive predictors to Measures, Shape and Space skills over time (Bull et al., 2008; Holmes et al., 2008; McKenzie et al., 2003).

9.4 Methodology

The methodology was detailed in Chapter 4 and summarised in Chapter 5. To briefly reiterate the variables measured are specified in the table below.

Table 30. Working memory and mathematics measures assessed.

Cognitive Domain	Task 1	Task 2
Verbal Short Term Memory (V-STM)	Word Recall	Nonword Recall
Central executive (CE-CWM)	Listening Recall (Verbal)	Odd One Out (Nonverbal)
Nonverbal Short Term Memory (NV-STM)	Mazes Memory (Visual Static)	Block Recall (Spatial Dynamic)
Performance Measures (Index of Nonverbal IQ)	Block Design	Object Assembly
Mathematics	Measures, Shape & Space Strand	

This chapter will later analyse the performance on the Measures, Shape, and Space Strand using hierarchical regression. In order to meaningfully scrutinize the data the raw scores on the Measures, Shape, and Space Strand were summed then a z score was calculated to standardise the data, the z scores were calculated as there were an unequal number of questions in the mathematics test relating to each individual strand (Table 31). This method ensured that the results for each strand would be comparable.

Table 31. Sample items from the Measures, Shape and Space Strand

Maths 5 (=10)	Look at these clocks. One of them shows four o'clock. Put a tick on it.
	Look at the teddies and the presents. They are on balances. One of the presents is **heavier** than the teddy. Which present is it? Put a tick on it.
Maths 6 (n=7)	Which is the tallest tree? Put a tick in the box below it. Which is the shortest tree? Put a cross in the box below it.
	One of these shapes has curved sides and straight sides. Put a tick on it.
Maths 7 (n=8)	The first box shows a letter T the right way up. In the second box it has turned through one right angle. It is about to turn through one more right angle in the same direction. What will it look like? Draw it in the empty box.
	Look at this balance. Is the parcel heavier or lighter than the sand? Or is it the same weight. Put a tick on the box with the right answer.

9.5 Results

9.5.1 Descriptive Statistics

Descriptive data for this strand is shown in Table 32.

Table 32. Descriptive Statistics for scores on Measures, Shape, and Space Strand (n=70)

	Measures, Shape, and Space			
	Questions (n)	Mean	Std. Dev.	Range (Min-Max)
Reception	10	5.89	1.76	7(2-9)
Year One	7	5.07	1.42	6(1-7)
Year Two	8	5.86	1.45	6(2-8)

9.5.2 Correlations

Table 33 presents the correlations between Measures, Shape, and Space Strand at each time of testing.

Table 33. Correlations between scores on the Measures, Shape, and Space Strand at each year grouping (n=70)

		1	2	3
1	**Measures, Shape and Space Strand Reception**	-		
2	**Measures, Shape and Space Strand Year One**	.46*	-	
3	**Measures, Shape and Space Strand Year Two**	.34*	.42*	-

* p=<.01

Reported in Table 34 are the one-tailed zero order correlations between each of the working memory measures and the Measures, Shape and Space Strand over the three year test period. The analyses statistically control for age in the subsequent regressions; as such it is not taken into account correlationally.

Table 34. Correlations between working memory measures, performance measures and the Measures, Shape, and Space Strand at each age range, one tailed (n=70)

	Measures, Shape , and Space		
	Reception	**Year One**	**Year Two**
Working Memory			
Word Recall	.35**	.42**	.21*
Nonword Recall	.07	.48**	.27*
Listening Recall	.36**	.40**	.37**
Odd One Out	.51**	.46**	.44**
Mazes Memory	.43**	.31**	.29**
Block Recall	.31**	.35**	.26*
Performance Measures			
Block Design	-	.12	.33**
Object Assembly	-	.28**	.49**

* p<.05, ** p<.01
n.b. Performance Measures were not assessed at Reception.

This early analysis identified that in Reception year each WM measure apart from Nonword Recall was significantly correlated with Measures, Shape and Space Strand performance *(all rs > .31, p*

< .01). In both of the subsequent cross sectional analyses all WM scores were significantly related to MSS (*all rs* $> .21$, $p < .01$). Furthermore, of the Performance Measures, in Y1 Object Assembly was significantly associated with MSS and by Y2 both Performance Measures correlated with MSS.

9.5.3 Regression Analyses

In assessing the amount of unique variance in performance on the Measures, Shape, and Space Strand both cross-sectionally and longitudinally using the hierarchical regression technique it was ascertained that in Reception year WM contributes a total of 24% of the unique variance in scores on this MSS strand (Table 35). This is after statistically accounting for age related variance, however it does not into take into account any variance provided by general Performance Measures as these were not assessed during this first school year.

In the Year One cross sectional analysis (Model 2) WM is accounting for 21% of the unique variance and this is above and beyond that variance from both age and Performance Measures. However in Year Two the table denotes that WM is not a statistically significant predictor variable following the examination of variance from age and Performance Measures.

In the final model (Year Two) it is evident that none of the individual WM components are statistically predicting performance on this Strand, but Block Design of the Performance Measures is independently influencing the mathematics outcome variable ($\beta = .30$, $p=<.02$) and Age is influential at all of the measured time points.

In unpicking the WM model into its separable components there is no specific pattern of independent WM variables influencing Measures, Shape, and Space. Reception year finds CE-CWM to be the better individual predictor variable, and in Year One while working memory as a whole is predictive of performance on this strand there is no significant independent predictor variable emerging from the WM measures.

Table 35. Hierarchical regression models predicting performance on the Measures, Shape, and Space Strand, controlling for age.

	Model 1: Reception Regressor: Measures, Shape and Space							Model 2: Year One Regressor: Measures, Shape and Space							Model 3: Year Two Regressor: Measures, Shape and Space						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1	.14	.12	.14	10.75**	.06	.06	.12	.09	.08	.09	6.94*	.01	.05	.02	.13	.11	.13	9.91**	.05	.03	.19
Age in Months																					
Step 2	-	-	-	-	-	-	-	.13	.09	.03	1.30				.31	.28	.18	8.77***			
Performance Measures																					
Object Assembly					-	-	-					-.04	.04	-.13					.01	.03	.07
Block Design					-	-	-					.02	.04	.09				-	.04	.02	.30*
Step 3	.38	.31	.24	4.08**				.34	.24	.21	3.13*				.38	.28	.07	.07			
Working Memory																					
Word Recall					.10	.07	.19					-.02	.08	-.04					-.04	.05	-.11
Nonword Recall					-.08	.07	-.14					.14	.08	.33					.07	.06	.16
Listening Recall					.08	.09	.11					.07	.07	.14					.03	.05	.09
Odd One Out					.15	.06	.31*					.05	.05	.16					.04	.03	.17
Mazes Memory					.10	.08	.15					.02	.05	.05					.03	.03	.12
Block Recall					.02	.05	.06					.03	.05	.10					-.02	.03	-.08
ANOVA	ANOVA [f(7,62)=5.45, p=<.0001]							ANOVA [f(9,60)=3.36, p=<.0001]							ANOVA [f(9,60)=.4.04, p=<.0001]						

Measures, Shape and Space 5 * p=<.01, **p=<.005, ***p=<.001; Performance measures not assessed
Measures, Shape and Space 6 * p=<.01, **p=<.002
Measures, Shape and Space 7 * p=<.02, **p=<.002, ***p=<.001

In the longitudinal model (Table 36) working memory is not identifiable as a significant predictor of performance on the outcome measure. However it was identified that both age *(R^2 Δ=.13, p=<.01*) and Performance Measures *(R^2 Δ=.18, p=<.05)* are statistically significant predictor variables of Measures, Shape, and Space when the mathematics outcome is measured two years following the initial working memory measures. Of these variables only Block Design is a significant independent predictor *(β = .33, p=<.05*).

Table 36. Hierarchical regression models predicting Measures, Shape and Space performance at Year 2 with WM measures measured 2 years previous, controlling for age and Performance Measures.

	Model 4 Regressor: Measures, Shape and Space						
Predictor Variables : Order of inclusion	**R^2**	**Adj. R**	**R^2 Δ**	**F Δ**	**B**	**SE B**	**β**
Step 1	.13	.11	.13	9.91**	.05	.04	.20
Age in Months at Y2							
Step 2	.31	.28	.18	8.77*			
Performance Measures							
Object Assembly Y2					.02	.03	.09
Block Design Y2					.04	.02	.33*
Step 3	.36	.28	.07	1.04			
Working Memory (R)							
Word Recall					.05	.04	.16
Nonword Recall					-.07	.04	-.22
Listening Recall					.04	.05	.08
Odd One Out					.02	.04	.07
Mazes Memory					.02	.05	.06
Block Recall					.00	.03	.01
ANOVA	ANOVA [f(9,60)=4.00, p=<.0001]						

*p=<.05, **p=<.01, ***p=<.001, a p=<.06

9.6 Discussion

In this chapter the endeavour was to shed light on the relationship between working memory and the Measures, Shape, and Space Strand. It was suggested that as the mathematical test is deconstructed into the separable, measurable "strands", so the influence of working memory upon these separable aspects can be unpicked. As there has been such a lack of research on this specific area this section was chiefly exploratory, although the hypothesis stated that CE-CWM and NV-STM may both have an influence upon this particular strand. This hypothesis was principally based upon the research of Holmes and Adams (2006), as their paper established correlationally that, central executive and nonverbal short-term memory were both related to Measures, Shape, and Space.

There is a reasonable predictive value for measures of CE-CWM in Reception but with regard to the nonverbal short-term memory and Measures, Shape and Space strand there was no evidence for a unique contribution being made by NV-STM to the scoring on this strand. This finding is in line with Holmes et al (2008) who, despite the strength of the correlational analysis, found that NV-STM scores did not predict performance in Measures, Shape and Space, only overall math. . Several research papers have proposed a key role for visuospatial memory in early general mathematical abilities (Holmes et al., 2008; McKenzie et al., 2003), but there is also a belief that this is a relationship that decreases over time. Age-related differences were found in the contribution of working memory as a whole to children's Measures, Shape and Space attainment. Overall working memory scores significantly predicted 24% and 21% of the variance in Measures, Shape and Space scores for the Reception and Year One age groupings respectively; however, by Year 2 that had fallen to only 7% of the variance in scores on that strand (n.s.). The inference from this is that working memory as a whole is considerably more important in the very early years of Measures, Shape and Space proficiency, but it appears that there is a significant decrement in the predictive value of working memory by the time the cohort reached the age of 7. At the early stages of the Reception year, where the child is using concrete representations to facilitate the completion of tasks on this strand, working memory may play a greater role (Geary & Burlingham-Dubree, 1989), yet as children gain more experience with mathematic procedures and facts, then more of this knowledge becomes embedded into long-term memory, and therefore CE-CWM may have less of a role to play in the solution of Measures, Shape and Space tasks. Consideration was given to the test questions for the Measures, Shape and Space strand and it was noted that they tended to require the student to manipulate visually presented information or use tiles to recreate or extend patterns. As an example, in one question children were required to discriminate between the tallest and shortest in a queue of line drawn people waiting at a bus stop. It is possible that the NV-STM was needed to a lesser degree to support the solution to these types of question due to the on-going availability of external visual support on the test paper or via the means of tiles or other concrete items.

The added complexity of the language of mathematics was also considered and this may be a mitigating factor as it is apparent that there is an influence of verbal short-term memory skills in Year 1. It was thought that this may be the point where the children have begun to form enough long-term representations to render CE-CWM no longer as important. It is entirely possible that the children in the youngest age grouping did not have concrete representations in long-term memory for concepts such as tallest and shortest, over and under, etcetera, and as such were utilising CE-CWM to greater effect in order to be able to attempt to provide a solution to the questions until this point.

Finally, it is worth mentioning that many of the remedial strategies that are encouraged for use when supporting children with difficulties in this particular curricular area of mathematics are visual and physical/tactile strategies. This, it is supposed means that the pupil’s MSS education will be facilitated by having concrete visual/physical representations of things such as clocks, 2D and 3D shapes, etcetera (DfES, 2001 - and this was frequently borne out by conversations with primary school teachers).

9.7 Summary

1. In summary, the present study puts forth additional evidence for a significant association between working memory and Measures, Shape and Space Strand in the two earliest years of formal education of children, with that role decreasing sharply at Year Two.
2. There was no evidence found that the curricular strand of Measures, Shape and Space is independently influenced by nonverbal short-term (visuospatial) memory. Additionally it is noteworthy that working memory does not appear to be an adequate predictor of MSS abilities longitudinally.

3. Importantly, the data extends our current understanding of the role of working memory in this particular aspect of children's curricular mathematics, with the inclusion of a younger child cohort, additional working memory measures, and a standardised curriculum based mathematical test.
4. Although still relatively speculative, these data offer an initial indication that the working memory processes supporting children's mathematics on this particular curricular strand change with age and the influence of working memory overall diminishes on this strand.
5. It is ascribed that this may be due to many of the features of the strand (features such as shapes, dimensions, units of measurement, and rules about patterns, etcetera) being committed to long term memory and as they are, stronger associations between LTM and WM are forged, thus relying less upon active working memory processes.

Chapter Ten

10 Discussion

In this final chapter the most important findings of the present thesis are summarised and an overview of the working memory contributions in each of the four measured mathematical strands is presented. There will be some discussion of the limitations of the research, some ideas about what else may be influencing curricular mathematics performance and aptitude in primary school children, and finally some directions for further research are presented.

During informal discussions with my working memory group colleagues we have often used the term “the maths mush” to describe some of the issues that have contributed to a mixed bag of findings about how working memory might influence, or predict mathematical attainment in typically developing children. Mathematics is a complex subject and even at the earliest level of primary school education comprises more than simple addition or subtraction tasks. It encompasses language, comprehension, general intelligence, attention, numerical and operand recognition and understanding. Mathematics also includes counting, number sequencing, negative numbers, different units of measurement such as money, distance or area, height and weight; recognising shapes and spatial references, problem solving, and interpreting data.

Just in those few sentences it should be clear why we thought of mathematics as a “mush”, and why it was considered important to try to tease apart the working memory factors that might impact upon performance on these vastly different skills. However, the notion of examining and identifying the impact of different working memory components and skills in numerical cognition is not a novel idea. It has been studied in different mathematical areas such as addition (Adams & Hitch, 1997; Barrouillet & Lepine, 2005), counting (Noël, 2009), transcoding (Camos, 2008), multiplication (Seitz & Schumann-Hengsteler, 2000) and problem solving (Andersson, 2007; Swanson, 2006b; Swanson et al., 1993; Swanson & Sachse-Lee, 2001) among many others.

However comparatively few studies have specifically considered curricular mathematics and the impact of working memory (Fuchs et al., 2010; Holmes & Adams, 2006; Holmes et al., 2008; Jarvis & Gathercole, 2003) and of those that have only three are grounded in UK curricular mathematics.

In the rest of this chapter provides a short overview of the research conducted in the present PhD thesis. Next, the main findings of this work will be integrated with the existing literature on working memory involvement in curricular mathematics. Limitations of the study will be discussed and finally, some avenues for possible future research will be presented.

10.1 General Conclusions

The overarching conclusion from this thesis is that working memory is significantly related to children's early mathematical attainment, both on overall mathematics and upon the four separable strands at each age range tested. The thesis also finds that working memory reliably predicts a significant amount of variance in scores on both overall mathematics and Calculation longitudinally. However it was also found that working memory is not a robust longitudinal predictor of Number, Problem Solving or Measures, Shape and Space.

Overall these findings support and extend research conducted by Gathercole and colleagues (Gathercole et al., 2003; Gathercole, Pickering, Knight, et al., 2004; Jarvis & Gathercole, 2003) where they argue that ability on working memory tasks is significantly associated with National Curriculum attainment in mathematics. These papers all discuss the strong relationship between measures of complex working memory and the Jarvis and Gathercole (2003) study identifies that nonverbal complex working memory is the task most significantly related.

It is believed that this thesis not only supports these findings but extends them by providing data that suggests that working memory measures, in particular complex working memory measures and a V-STM measure predict performance on a UK curricular mathematics test, above and beyond a cognitive performance measure (IQ) and age. This is with specific regard to children of a considerably younger age range than those in the cited studies. As in Jarvis and Gathercole (2003) this thesis argues that in general it is a measure of nonverbal complex working memory that is overall the better predictor of performance on a curricular mathematics test. Furthermore this thesis establishes that complex working memory is predictive of curricular mathematics “strands” as the mathematics test used was directly associated with the school curriculum.

In Chapter 5 it was found that working memory can quite reliably predict between 17% and 36% of the variance in mathematics scores at each year grouping after age and Performance Measures had been statistically accounted for. Both CE-CWM and verbal short-term memory emerged as good predictors, and previous research has suggested that verbal short-term memory skills may support the retention of verbally presented mathematics information (Adams & Hitch, 1997). The mathematics test that was presented to the participants was in the form of a response booklet that contained a visual representation of each question (e.g. a sum such as 14 – 2, or an image such as people in a bus queue), but each question had to be verbally read out by the experimenter, and to some extent these findings show agreement with this idea. However this is only clear at the Year One grouping where Nonword Recall is a significant independent predictor variable.. It is thought on this basis, that at this age grouping children are at an age where they are beginning to spontaneously utilise subvocal rehearsal to support the retention of mathematics information (Fischer, 2008; Flavell et al., 1966; Gathercole et al., 1994). These data also provide some evidence that working memory ability may support children’s mathematical attainment independently of the contribution of a higher order construct such as IQ (Alloway, 2009; Kyllonen & Christal, 1990). Furthermore these data also support and extend the research of Holmes and Adams (2006) by introducing a younger cohort of children and increasing the number of working memory measures in the procedure.

At the Reception year grouping a visual measure of the central executive is the only significant independent predictor, at Year One a verbal short-term memory measure and both central executive measures are significant independent predictors, and at Year Two the verbal complex working memory measure emerges as the lone significant independent predictor variable. In considering this data some conclusions were reached; At the Reception age group it is likely that the significance of Odd One Out (CE-CWM) is apparent as **all** mathematics is challenging at this age. It is well documented that children with poor working memory profiles experience problems in carrying out complex tasks and at the first age grouping in this study the children are aged 4 to 5 years old, and will not really have encountered mathematics in a structured way before. At this point they are being introduced to not only numbers, but also operations, magnitudes, negative numbers and place value, to name just a few. In any situation where novel tasks are being introduced one would naturally expect that children would require significant cognitive resources to be able to process the information and this school situation is no exception. Children may be familiar with numbers as verbal concepts and also as concrete representations (such as when playing with blocks) but the strong likelihood is that most children aged 4-5 will not have encountered a great deal of written mathematics. Armed with this understanding it can be seen how the processing of mathematical information will be heavily reliant upon complex working memory resources as the child seeks to process, understand and commit to long-term memory the information being provided.

At the second age grouping however, it is interesting that Nonword Recall also becomes a significant independent predictor or mathematics performance alongside complex working memory. This age group are 6-7 years old and at this age children are spontaneously learning how to subvocally articulate information (Gathercole et al., 1994) and it is believed that this novel skill is influencing their performance in maths in this study. At this age range children will have experienced one year of full time school education and should have committed a great deal of mathematical information to long-term memory, thus increasing automaticity of facts and potentially freeing up some of the previous central executive - complex working memory resources. The Nonword Recall task is thought to minimise support for recall from long-term memory and is talked about as being a pure measure of phonological processing and it is suspected that the role of phonological processing in this age grouping is allowing those children who have successfully committed information to LTM to use a direct recall strategy more efficiently via V-STM.

In the third testing phase the children were aged 7 and the data shows that CE-CWM is again a significant independent predictor variable, however at this time point it is the verbal measures of complex working memory that is significant. Whilst CE-CWM and V-STM measures are separable they do correlate and this finding may be evidence that the children are beginning to integrate the verbal storage and processing aspects into their mathematical learning in a more effective way. It is possible that the questions in the mathematics test at this age range have an effect as they are increasing in complexity, and many have a strong verbal element.

10.2 "Strand" Specific Conclusions

Throughout this thesis four mathematics "strands" that are consistently referred to in the Department for Education[5] curricular documents have been described (see chapter 3). These strands (Number, Calculation, Problem Solving and Measures, Shape, and Space) were examined in order that attempts to unpick the influence of working memory upon those separable skills could be made.

The Number Strand reported in Chapter 6 is fundamental as a building block for mathematical learning in general and it was revealed that working memory significantly predicted between 14% and 25% of the unique variance in scoring on the dimension of "Number" in the year-on-year analyses. However it is apparent that over time the influence of working memory decreases on this strand, and this is thought to be as a result of many mathematics facts, such as number bonds, counting, and number sequences being committed to long-term memory. There is also an assumption being made that these facts are regularly retrieved from LTM, thus strengthening the memory traces and increasing automaticity.

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[5] formerly named Department for Education and Employment and Department for Education and Skills

Calculation (Chapter 7) seems to be the natural logical successor to Number, and this study identified that working memory consistently predicted about a quarter of the unique variance in scores on this strand, regardless of age and influence of performances measures that indexed IQ. This also stood up to the longitudinal test and it seems clear that working memory is a robust predictor of Calculation performance.

The waters become a little muddier when considering the next two strands (Chapters 8 and 9). Problem Solving (Chapter 8) was a strand that initially appeared to be quite straightforward to examine. Previous studies have referenced central executive ability as being important in this facet of mathematics (Andersson, 2007; Kail & Hall, 1999; Lee et al., 2004; Swanson, 2006b) however the pattern of data presented in this thesis is somewhat unclear on the influence of working memory upon this strand. Firstly the regression analyses showed evidence that in Reception WM can significantly predict 24% of the variance in scores; in Year One 11% (ns); in Year Two 15%.

One of the issues clouding this strand is that Problem Solving in the curriculum is not as clearly specified as it is in the psychological research. In the psychological literature it is typically defined as arithmetic word problem solving, the format of which comprises of a word based problem with multiple steps to reach an answer (most often including a calculation), and sometimes containing irrelevant information as an extra attentional demand. However in the UK curriculum the Problem Solving strand covers a very broad area, and arithmetic word problems are only a very small part of the strand. To this end the questions relating to the Problem Solving Strand were examined in more detail and it was found that in Reception, there were only five questions and of those, only one took the form of a multi-step problem. In Year One a floor effect in the data was noted, and as such this is not a reliable measure at this age range, with this cohort. It became apparent that the children in the cohort may not have reached the time in that particular school year where they had been taught monetary values and the vast majority of items on the Mathematics 6 relating to Problem Solving contained problems pertaining to money/coins.

There was some limited past evidence of a strong involvement of calculation in Problem Solving (Andersson, 2007) and the thesis sought to explore this in further detail. The inclusion of Calculation into a mediation model helped to establish the extent that Calculation mediated performance upon this strand (Andersson, 2007). Given that Chapter 7 informs us that working memory can reliably predict around a quarter of the variance in scores on Calculation it was important to model this statistically with reference to the influence of working memory upon Problem Solving, via Calculation. Our key finding here was that Calculation fully mediated the relationship between working memory and Problem Solving, meaning that in this study, working memory and Problem Solving are only related because of working memory's effect on Calculation. Some caution should be used in interpreting the model as the analyses only modelled the final year of data collection and not each year, it is still a useful extension to the findings of Andersson (2007).

In Chapter 9 the Measures, Shape, and Space data showed that in the first two years of testing performance on this aspect of mathematics could be predicted by working memory with about 20% of the variance being accounted for, and both CE-CWM (Reception year) and verbal short-term memory (Year One) were significant independent predictors, a very similar pattern of data to that of mathematics overall. It was expected that visual short-term memory may be influential on this strand, but this was not shown to be true. In fact across all of the strands no evidence was found for a significant role in mathematics for nonverbal short-term memory. In order to rationalise this finding it was necessary to consider the questions in the mathematics test for this strand, and it was discovered that many of the test items needed the child to manipulate visually presented information or use tiles to recreate or extend patterns. Another example of a Measures, Shape, and Space question is that the child was required to discriminate between the tallest and shortest in a queue of line drawn people waiting at a bus stop. For this the argument is put forth that the on-going external support of visual or concrete items meant that the child did not need to utilise visual short-term memory resources, hence the lack of a relationship between NV-STM and Measures, Shape, and Space.

10.3 Associations between Digit Recall and Mathematics

In Chapter 5 the thesis examined a methodological problem that has been previously discussed (Holmes & Adams, 2006; Lehto, 1995; Passolunghi & Siegel, 2001), and then to some degree refuted by Alloway (2007). This was the matter of the excessively strong reported associations between Digit Recall (and possibly all numerically based working memory tasks) and mathematical performance. The idea being that it is possible that working memory and mathematics are so commonly linked, and possibly mistakenly so, because the measurement of both mathematics and numerically grounded working memory tasks involves either number processing or direct access to numerical information. It was felt that this issue had never been adequately resolved in a typically developing child population in the previous literature, and from the study design in this thesis there was an opportunity to collect and analyse this data over a three year period.

Firstly the correlational data was considered, and at each cross sectional time point the correlations between Digit Recall and Mathematics were strong. The only other working memory variable with similarly strong correlations was Odd One Out (CE-CWM).

Then the difference in the strength of correlations between a numerical V-STM task (Digit Recall) and mathematics scores was analysed; and two word-based V-STM tasks (Word Recall and Nonword Recall) and mathematics scores. The difference between correlation coefficients was calculated based on the value of the coefficients and the sample size (Cohen & Cohen, 1983). The results were unfortunately not wholly conclusive, as it was found that in only two out of the three years studied, that there was a significant difference between the effects of Digit Recall and word based working memory tasks on the mathematics outcome (Chapter 5). It was felt that this was sufficient evidence to justify the exclusion of Digit Recall in the further statistical analyses in this study. In conclusion it is the position of this thesis to recommend that numerically based working memory tasks should not be used when examining mathematics, or the results should be interpreted with more caution on the understanding that these measure of working memory and mathematics are both tapping into a common numerical process.

10.4 The absence of a strong relationship between NVSTM and Curricular Mathematics

This thesis talks frequently about the similarities between this and Holmes and Adams (2006) where they find a small but statistically strong relationship between NVSTM and curricular mathematics, however this thesis provides no strong evidence for such a relationship.

Firstly we should consider the key differences between the two studies. The children in the Holmes and Adam study are older than those here and as such are at a different stage in their mathematics and working memory development. It has been pointed out quite frequently that we cannot necessarily directly compare the results of older and younger children, and in the simplest form this could be the driving force behind the disparity in the results between this thesis and its closest precursor study. The Holmes and Adams paper uses the same working memory measures as this thesis, however there were problems described in Chapter 4 with the adoption of the Automated Working Memory Assessment task Mazes Memory. Given these issues this study used the paper and pencil version from the Working Memory Test Battery – Children. The AWMA and WMTB-C versions of this task were highly correlated and thought to be measuring the same construct. I will discuss the issues with the Mazes Memory Task in 10.6.1 as a weakness in the study so will not go into great detail in this section.

Aside from the practical reasons for the absence of a relationship between NVSTM and mathematics there are some other perspectives to consider. A recent study by Caviola et al (2012) argues that mathematical problems that are presented vertically are more affected by load on visuospatial sketchpad. This was considered as a possibility as there are very few questions in any of the Mathematics 5-7 tests where the arithmetical problems to be solved are presented vertically. Assuming the items presented in the appendices of the Holmes and Adams study are highly representative of the whole of the mathematics test they designed, then there is little indication that they presented items vertically too. However the children in both the Caviola et al study (Caviola et al., 2012) and the Holmes and Adams study are of a similar age and exhibit a similar relationship between NVSTM and mathematics, therefore I cannot reject or confirm the idea that presentation modality of the mathematics problems is the root of the lack of a relationship between the variables.

Another possibility for the lack of a relationship is found in Simmons et al (2012), where they show that nonverbal short-term memory is predictive of number writing skills. They suggest that children formulate and store visual –spatial representations of numerical information in order to transcribe them. Addition accuracy was not similarly influenced by visuospatial measures in the Simmons et al study and they also discuss presentation format as perhaps being an important aspect in the relationship between NVSTM and mathematics as the addition problems were verbally presented and they seemed to require the storage and processing assistance of the central executive. This may be applicable with regard to the children in this study also as the Mathematics 5-7 tests required the questions to be read out to the children. They did have the benefit of a visual representation of the questions, but it is possible that the concurrent aural presentation may be a confounding factor. In summary, while no strong conclusions can be drawn about the absence of a predictive relationship between NVSTM and mathematics in this thesis, there is some other research than indicates that presentation modalities might be a key factor in explaining these data.

10.5 Wider Practical and Theoretical Implications

10.5.1 Predictive Value of Odd One Out Task

Few studies have used the Odd One Out task (or other nonverbal CWM measures such as Mr Blobby/Mister X or Spatial Recall) instead favouring the more common Listening Recall task and Backwards Digit Span but a number of studies have identified that CWM is an effective early predictor of classroom performance in a variety of tasks, (Berg, 2008; Bull et al., 1999; Geary et al., 1991; Holmes & Adams, 2006; Passolunghi & Siegel, 2001; St Clair-Thompson et al., 2010; St Clair-Thompson, 2011) and this study extends this finding to take account of performance in younger children, and over a long period of their early education. It is thought that one reason for the predictive value of this task in particular is that children did not seem to use any verbal strategies to complete the task and it appears to be quite a "pure" measure in that respect. Furthermore, from the correlational analyses we can see that where Digit Recall has strong correlations with mathematics, the same can be seen for Odd One Out, yet this task has no numerical basis. However there appears to be something of a developmental shift between NV-CWM and V-CWM with early mathematical attainment being best represented by Odd One Out (NV-CWM) and later mathematical performance being better predicted by Listening Recall. There are some possible explanations for this. Firstly methodologically, the Listening Recall task is a very hard task to use with younger children and as such it might not be the most appropriate measure to use at the age of four to five years old and performance on this task was at a low level, particularly in Reception. Conversely the children engaged well with the Odd One Out task, and performance was at an acceptable level comparable with standard scores. Furthermore, theoretically there is the likelihood that previously discussed developmental aspects relating to the emergence of subvocal rehearsal has had a direct influence on the efficacy of the verbal complex span tasks. It could be argued that the ability to rehearse to-be-remembered material aids the maintenance of verbal information and in turn this may influence updating and attentional refreshing in central executive tasks. There is also the possibility that this is somehow related to the episodic buffer in which verbal representations of task relevant information are held either in a functionally separate verbal short-term store (Baddeley, 2000) or as an activated subset of verbal information stored in long-term memory (Cowan, 1995). This is of course speculative but could be an interesting line of future advancement. There is evidence that whilst children use subvocal rehearsal for content of a verbal nature, it is apparent that they do not tend to do so for visual/pictorial information until around the ages of seven and eight (Halliday, Hitch, Lennon, & Pettipher, 1990; Hitch & Halliday, 1983) and this thesis can see some evidence of this, particularly in relation to chapter 7 where Odd One Out becomes predictive of Calculation again, alongside Listening Recall, perhaps suggesting that developmental integration is occurring at this time point.

Lastly, taking both age related variance and variance pertaining to performance measures, the data also suggest that CE-CWM is not an equal substitute for IQ, but appears to represent a dissociable cognitive skill that has unique links to learning outcomes in curricular mathematics.

10.5.2 Training Working Memory to Improve Curricular Mathematical Performance

Alloway and colleagues (Alloway, 2009; Alloway & Alloway, 2010) argue that working memory has been found to be the number one predictor of academic achievement in both literacy and numeracy with the claim that working memory is better than traditional measures of intelligence (Alloway, 2009). Evidence suggests that working memory scores have none of the biases that are found in IQ tests, and are not related to socioeconomic indicators like cultural background (Campbell, Dollaghan, Needleman, & Janosky, 1997). The typical developmental trajectory of working memory follows a relatively stable linear growth path throughout childhood (Alloway et al., 2006) and the present thesis research supports this.

The idea that working memory can possibly be trained has been leading the research into working memory and cognitive skills since around 2009 (Alloway, Bibile, & Lau, 2013; Dunning, Holmes, & Gathercole, 2013; Holmes, Gathercole, & Dunning, 2009; Holmes et al., 2010; Klingberg, 2010; St Clair-Thompson et al., 2010; von Bastian & Oberauer, 2013). However there is also a swathe of literature that counters, disputes or tempers the notion that training improvements in working memory can be maintained long-term, or can provide useful transferable gains in performance on scholastic or other cognitive measures (Hulme & Melby-Lervåg, 2012; Melby-Lervåg & Hulme, 2013; Morrison & Chein, 2011; Redick et al., 2013). One possible reason for the vociferous decrying of training gains/plasticity/transfer could be that many of the tools marketed to train working memory have a distinct price tag attached alongside in-house research, such as Cogmed (Klingberg et al., 2005) and Jungle Memory (Alloway & Alloway, 2008) (to name just two from a great deal of working memory training software). It seems reasonable to be sceptical about increased working memory skills, and transferable gains across scholastic and cognitive domains when faced with financial rewards for the authors and publishers of such tasks. In fact the recent meta-analysis by Melby-Lervåg and Hulme (2013) argues quite strongly that whilst training did reliably improve aspects of working memory in the short-term, there was a lack of convincing evidence to support the idea that these improvements could be long lasting, or generalizable to other cognitive skills.

When Holmes and colleagues (2009) took a training programme into the curricular arena, working with children with low working memory spans and conducting adaptive training with them, with some remarkable findings. They found that the training improved the children's working memory spans to bring them to within a normal range, and that those gains persisted (and even continued to improve) in post-test six month review. St Clair-Thompson et al (2010) obtained a similar effect, showing modest improvements upon some working memory components via training, but they found no discernible improvements in performance on standardised tests of reading and mathematics.

In a recent article Gathercole and colleagues (Gathercole, Dunning, & Holmes, 2012) caution against a knee-jerk "throwing the baby out with the bathwater" with regard to working memory training, and this piece highlights some of the methodological difficulties that are faced that using the programs available in an applied setting. Furthermore, whilst the transference to other cognitive skills may not be evident, it is feasible to suggest that children with specific deficits in working memory who achieve gains in their working memory from cognitive training might achieve other, non-cognitive benefits. For example there is a possibility that improving working memory might impact on self-esteem in a classroom setting as they may feel better able to cope with the cognitive demands, or perhaps anxiety in the classroom or disruptive behaviours might decrease. It would appear that this is not something that is adequately addressed in the current literature on the topic and whilst this thesis acknowledges the speculative nature of these ideas it could prove important to quantify them with research. Some further work indicates that, in reality, this type of research is still very much a work in progress (Shipstead, Hicks, & Engle, 2012) and this thesis also takes that view.

10.5.3 Strategies for supporting children with working memory deficits and improving performance in the classroom

A consistent finding throughout this research is that complex working memory is a significant independent predictor of curricular mathematics, and in particular Number and Calculation. Both Number and Calculation have a substantial role to play through a child's schooling in mathematics, and it is thought that one of the most important implications of this research is that there is a potential for more carefully targeted support and working memory training for children with deficits in complex working memory. Most importantly these findings relate to typically developing children, without diagnoses for dyscalculia, attentional deficit problems, specific behavioural problems, or language impairments. The children in this study are ordinary children, quite representative of the wider population, yet some of them present with poor working memory profiles and are likely to struggle with complex instructions and in particular tasks that require on-going storage and processing functions.

Some researchers have put forth recommendations for teaching strategies to facilitate learning in pupils with poorer working memory profiles (Gathercole, Lamont, & Alloway, 2006; Gathercole & Alloway, 2004). They suggest that it is important to recognise working memory problems. They put forth that teachers should be aware that the kinds of problems that children face will be incomplete recall, a failure to follow a sequence of instructions and task abandonment. A child presenting with these issues could be perceived as being uncooperative, so it is important to evaluate the possibility that there is an underlying cause that is not related to behaviour management. This thesis highlights the fact that the children in the present study were very young children, at the start of their school life, and the implication of this is twofold. It is felt that it is possible identify children who have deficits in working memory at a very young age with one quickly administered working memory task, Odd One Out. Potentially this means that teachers will have the opportunity to provide specific targeted interventions that will not only improve their working memory capacity, but may also have a significant impact upon their curricular mathematics. There can only be benefits to early identification as the relationship between working memory and both overall mathematics and Calculation, seem to remain largely stable. Early identification could potentially reduce the risk of more significant problems later in school life for these children. There is a very small body of research that has attempted to evaluate methods for improving long-term academic attainment (not specifically mathematics) where working memory has been identified as being problematic. Interestingly the findings of Elliot et al (Elliott, Gathercole, Alloway, Holmes, & Kirkwood, 2010) show that neither a classroom based working memory approach, or precision teaching improved WM scores, or academic gains, but there was some evidence that that the teachers who had adopted recommended teaching strategies appeared to be associated with better academic performance. It seems that teachers who are sensitive to the cognitive needs of children with WM deficits are more effective at overcoming some of the issues around working memory in the classroom and this supports Gathercole and colleagues' assertions that recognising working memory failures is important.

Gathercole and colleagues further recommend that teachers should monitor a child who is presenting as having working memory problems. The child should be strongly encouraged to seek help when needed and they may not be able to work as independently as children with typically developing working memory profiles. They will likely need instructions to be broken down into smaller chunks, and have them reiterated to them. Furthermore, teachers should evaluate the demands of the task when planning learning activities and integrate opportunities to reduce working memory load.

As discussed earlier in the thesis, curriculum documentation endorses the idea that children with mathematics difficulties should be given support by means of tactile and visual aids (DfES, 2001). This thesis provides evidence to support this strategy based upon poor working memory skills rather than poor mathematics skills. The mathematics test used was heavily populated with physical aids which appeared to reduce load on NVSTM. Where other studies have found a relationship between NVSTM and mathematics, this thesis finds no evidence for this relationship and it is believed that this is due to the supporting visual and tactile aids.

There is evidence that young children do not engage in subvocal rehearsal, (Gathercole et al., 1994) and the suggestion is that this skill emerges spontaneously at around the age of seven (Gathercole, 1998; Gathercole, 1999). This thesis supports this finding, and as such it is recommended that cognitive strategies that employ this skill should be avoided before Year One and Year Two in school.

10.5.4 Theoretical Considerations

10.5.4.1 Fractionation of the central executive and links with nonverbal STM

Fractionation of the central executive is not a new idea, having been discussed many times (Baddeley, 1996b, 1998; Fournier-Vicente, Larigauderie, & Gaonach, 2008; Tsujimoto et al., 2007). This study does not find any strong evidence to suggest that the nonverbal and verbal working memory components that were measured are domain specific, but the close links between the nonverbal working memory measure and nonverbal short-term memory measures should be made note of. The correlational data shows that there are strong relationships between Odd One Out and both of the nonverbal short-term memory measures. In particular the relationship between Odd One Out and Block Recall is indicative that these measures are likely tapping into a similar construct. These data are somewhat linked to two previous studies (Jarvis & Gathercole, 2003; Shah & Miyake, 1996) where there is some evidence for separate pools of resources for verbal and nonverbal working memory, however this is rather speculative.

10.5.4.2 Nonverbal short-term and working memory and curricular mathematics

A number of other studies have found associations between measures of visuospatial memory and mathematics (De Smedt, Janssen, et al., 2009; Holmes & Adams, 2006; Holmes et al., 2008) whereas this thesis finds no evidence for such a relationship. However his thesis brings forward an argument that in young children nonverbal complex working memory span may be functioning more effectively in place of nonverbal short-term memory. It is believed that this is as a result of the on-going concrete visual and tactile support that is provided in the mathematics tests used. It is thought that as nonverbal short-term memory is not needed, that resources are being directed via nonverbal complex working memory instead. Hence the relatively strong relationship between the central executive and mathematics, and in particular the Odd One Out Task. In the Holmes and Adams study (2006) where curricular mathematics was examined, the mathematics test they used had no visual support in the form of external tokens or pictures and it is thought that this new finding has implications for future studies that might examine the links between mathematics and visuospatial memory. It is believed that it should be a consideration that curriculum mathematics is not necessarily analogous to mathematics when measured without concrete visual or tactile support.

10.6 Strengths, Weaknesses and Limitations of the Study

10.6.1 Validity and reliability of measures

10.6.1.1 Working Memory Measures

The validity of measurements is always open to question. Encouragingly, the tests used to measure working memory capacity are well established and there is great confidence that they do in fact, measure these components correctly and data to this effect is reported in Chapter 4.

Obtaining inter-item reliability on the current AWMA data would be severely problematic due to the way the AWMA is conducted. For example, in Word Recall each "span" has 6 levels. At level one, there is one word to be recalled, at level two there are two words to be recalled and so on. The task terminates if two items are incorrectly recalled. For instance, the participant could fail to recall the first and third items, or they could get the first four items correct. There are a number of different permutations of this at each span level. As such it was not considered appropriate to conduct any reliability at an item level.

However in this thesis a concern has arisen on two levels about the Mazes Memory task. Firstly performance on the task was lower than would be anticipated in a typical population and secondly Mazes Memory is not correlating with the secondary nonverbal STM measure, Block Recall. Reflecting on the issue of poor performance on the task, the statistical data were reconsidered, and there were no specific problems with the quality of the data, and the developmental path of year on year growth is typical. This implies that this particular sample were simply less adept at this task when measured against a typical normative sample.

When considering the lack of an expected correlation between the two nonverbal measures at Year One it is possible that the different presentation format to the AWMA tasks has had an effect upon the correlational relationships between the WMTB-C measure and the AWMA measure. However, this seems unlikely given that the tasks were all modified from the WMTB-C (Pickering & Gathercole, 2001). Examination of the correlations showed that the Block Recall task was only correlated with Mazes Memory in Reception, but by Years One and Two it seems that it has an increasingly strong relationship with the phonological and working memory measures. This leads to the tentative suggesting that something different is occurring in how the children fulfill the Block Recall task. It is tentatively suggested that the children are using visual resources in the early year of testing but in Years One and Two it seems likely that the children are attributing a phonological code to the Block Recall task thus the same cognitive processes would not be involved. When testing the participants this idea that they were giving a visual measure a phonological representation was apparent, and it was clear that they would frequently attach a number to the blocks to be recalled as a strategy to help them remember the sequence, or they would vocalise sounds like "duh, duh, duh.." each time a block was tapped.

It should also be discussed that at the earliest age range the scoring on the Listening Recall was relatively low with a small range (0-3 items). Although this task has been tested with children aged 4-5 years old in previous work, the administration of the task with small children is quite difficult and some children seem unable to comprehend the rules and processes that they need to follow. This thesis suggests that this task should be used with caution in children as young as those in the Reception year grouping.

10.6.1.2 Mathematics 5, 6, and 7

It may have been helpful to gain individual mathematics SAT scores for each child, however the schools were reluctant to provide that data. If the opportunity to correlate the scores on the Mathematics 5-7 test with the SATs had been available then this may have given an extra dimension of reliability to the Mathematics 5-7 tests with regard to the mathematics curriculum. The Mathematics tests administered had been examined for internal consistency reliability and had been found to reliable using the Kuder-Richardson 20 formula (manual of NFER-Nelson, 2001). Furthermore test-re-test reliability had been conducted and the correlations were .78, .78 and .79 for Mathematics 5 through 7 respectively.

A number of issues became apparent when analyzing the 'strand data'. A concern that arose in

Year One was that there was something of a floor effect in the Problem Solving strand data. On scrutiny of the items in that strand it was clear that a large proportion of them were related to the interpretation and manipulation of money. It is thought that this effect may have arisen because of the time of the testing sessions in the year. Children were tested between January to April in each year and it is likely that in Year One the children had not yet been taught about money. The Mathematics 5-7 tests have items that span the whole of each school year and as such it is possible that this effect was also mirrored to a lesser degree in some other items within the test too.

It should also be noted as a weakness that the number of items for each strand varied, this is discussed in each strand chapter, and despite attempting to control for this by standardising the scores on each strand it may have still been problematic, particularly where the number of items was very disparate.

A final thought on the Mathematics 5-7 test is that one cannot ignore the strand crossover that occurred on a number of items. For example in Appendix F the table identifies all the items at each age range. In Mathematics 6 (Year One) the majority of the questions are linked to another strand. E.g. Question 18. How many pairs of socks are there? The pictorial representation was of six identical socks. The curriculum links sheet supplied with Mathematics 6 suggests that this question links to Number (counting, magnitude judgment) and to Problem Solving (knowing that two socks equals a pair) as well as calculating that 6 (individual socks) divided by 2 (the multiple of a pair) equals 3. Of course there are other ways that the correct answer could be achieved too.

These considerations do lead one to suppose that creating a mathematics task that included only items that were more separable would be beneficial in the analysis of the strands. Using this test meant trying to shoehorn an applied measure into a theoretical model and this can be viewed as a compromise. However it is felt that the real-world, applied nature of this test does also provide a benefit in that it matches the curriculum in form and structure.

10.6.1.3 Performance Measures

A clear limitation of this thesis is a methodological issue. In the first phase of testing, when the children were in Reception classes, the study did not account for intelligence (Performance Measures). There were significant time constraints that meant that the omission of those measures (Block Design and Object Assembly) from the first wave of testing was necessary, and in hindsight this is something of an error. In terms of the impact of this upon the project, there is one key area in which it is believed that it may have had an effect. In Chapter 8 where the Problem Solving Strand is examined, the data for the Reception grouping identifies working memory as a significant predictor variable, but in the subsequent two cross-sectional years working memory is non-significant. The rationale is that this significant association in Reception year is a misappropriation of some of the shared variance between Performance Measures and working memory measures. This however, is not to say that working memory is not influencing Problem Solving. It was demonstrated in the analyses that Calculation is a significant predictive factor, and working memory contributes around a quarter of the variance in Calculation therefore it is believed that working memory is indirectly influencing Problem Solving via Calculation.

There are also some other measures that with hindsight would have benefitted the project. The study may certainly have profited from having a measure of vocabulary, reading ability or comprehension. The reason for this is that mathematics is a subject that not only has its own vocabulary (with words such as “algebra”) but also it borrows words from normal everyday language and this can cause confusion. Some examples where this might be an issue are the words that are used to imply the same mathematics operation; add, plus, more than; subtract, minus, take away (adapted from Chinn, nd). Chinn cites “take away” as confusing as in every day discourse we could use that to describe food purchased to take off the premises of the establishment. Furthermore mathematical vocabulary confusion can also arise from the way we use these words in a word problem. Chinn (nd) provides a good example of the confusing way words are used in mathematics. 'Mark has three more pens than James. Mark has ten pens, how many pens does James have?' (p. 4). In that sentence the 'three more [than]' is a phrase that would typically be associated with addition, yet the problem to be solved is a subtraction problem. However, for the most part the tasks used in the study did not require the child to be able to read, so it is possible that the impact of not testing this aspect is minimal.

10.6.2 Sample Size

A greater sample size may have been more sensitive to differences in all measures used, but in particular the separable "strand" mathematics scores. Statistically speaking the criteria for conducting the analyses that were deemed to be the appropriate ones was fulfilled, but it is accepted that the number of participants was at the lower end of the acceptable criteria, and this could have influenced the resulting data

Attrition is a related issue that may bear some scrutiny. In this study, children who withdrew before the conclusion of the study or who had incomplete data for any other reason were eliminated from the final analyses. The reasons for eliminating these pupils from the final study were largely pragmatic. As a part-time postgraduate student researching in schools time was a huge constraint, particularly at the last phase of testing, where the children were reaching higher working memory span levels and performance measure levels thus testing sessions were taking considerably longer. Furthermore there was only a limited window for data collection with regard to the length of time that the schools would allow a researcher to be there. In hindsight, perhaps with different statistical modelling techniques, children for whom the data are not complete could still have been included in the model. This could have increased the overall sample size and may have enabled the models to be more sensitive to differences between the groups.

10.6.3 Growth Analyses

This research seemed to lend itself well to analysing growth; however a number of issues with the Mathematics 5-7 test indicated that the data would not be particularly reliable. The test uses a scaled score, converted to a progress score to measure growth and the initial Mathematics test ran from Mathematics 7-14, with Mathematics 5 and 6 being developed at a later date. A constant was added to the scale score range on Mathematics 7-14, so that all values in the scaled score would be non-negative. The manual states that it was necessary to keep the progress scores on the same metric when they introduced Mathematics 5 and 6 in order to avoid rescaling tests that had been in use for a number of years. Therefore a number of scaled scores at the low end of the raw scores are negative values. The manual states that progress scores can only be estimated rather than calculated exactly and caution should be used when using progress scores. On this basis analysing growth for the purposes of this study was considered to be problematic, however this should be an avenue for future research.

10.6.4 Factors beyond the control of the study

A potential problem that became evident during the course of the research was that the information and guidelines in the education arena alters so quickly. When the project began the government had released guidelines for primary school educators in the form of The National Curriculum, (DfEE & QCA, 1999b) and The National Numeracy Strategy (DfEE, 1999) and many evaluations of the efficacy of these guidelines were being undertaken (Ofsted, 2000a, 2000b, 2000c, 2001, 2002). Subsequent to these evaluations the Primary Framework for literacy and mathematics was introduced (DfES, 2003a), this framework document was somewhat less prescriptive than The National Numeracy Strategy in its recommendations to teachers. Most recently of all the National Strategies website closed in June 2011 and as such access to some of the online resources is now impossible, as only those deemed popular and useful have been preserved in the National Archives website (DfES).

Had the “strands” altered significantly then this could have been a much greater issue for the study, but the commonality of the strands runs right through all of the current documents regarding the mathematics curriculum, so they are still clearly identifiable. In real terms the effects of this have been quite minimal on this project, but the rapid change in policies is definitely a factor to be aware of when considering future research into any subject within the UK curriculum.

10.6.5 Other Factors Affecting Learning

There are always going to be a number of other factors that will influence learning on any subject. Khan and Weiss (Khan & Weiss, 1973) identify a model whereby they indicated the factors that influence attitudes to learning (see Fig. 8.) and in many aspects these factors can also be applied to learning itself.

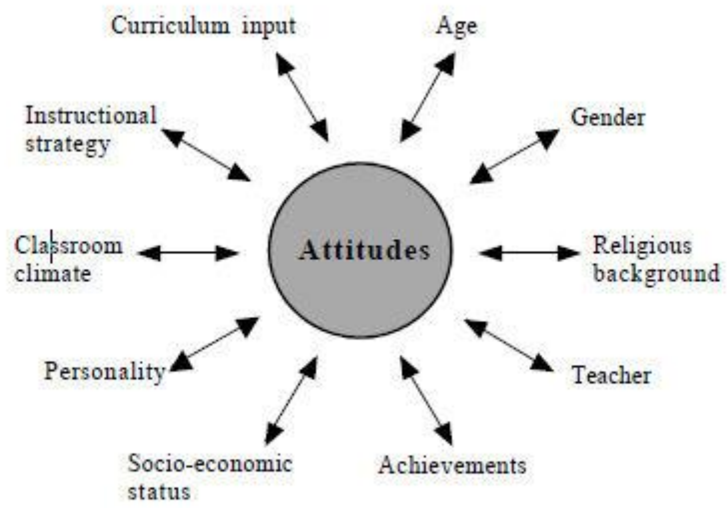

Figure 8. Factors influencing attitudes [to learning] Khan and Weiss (1973)

The study tried to address some of the issues that Khan and Weiss identify as being important in attitudes towards learning; the study took account of age, gender, and curriculum input, in the design. Classroom climate can be interpreted in a number of ways, it could be taken as how secure and happy a child feels within the classroom, with its peers, with its teacher, or it could be physical environment factors such as temperature, furniture and resource provision. Again this could not be stringently controlled. The working area for the testing phases was in general a quiet area, typically the library, but on occasion it was not possible to use these quiet areas and flexibility had to be employed, also the physical environment in one school was considerably more pleasant than the other. This may have influenced the study in so far as the children in the school with the much improved physical environment significantly outperformed the children in the other school in the first two years of the study.

The factors of “instructional strategy” and “teacher” are both factors that would be impossible for experimenters to govern in school circumstances, but curriculum input may mediate these factors to some degree. The curriculum design at the time of testing was somewhat prescriptive and teachers had to complete a number of processes in their teaching strategies (such as providing alternative mathematics strategies and time for consolidation of learned facts). Some of these factors were beyond both the scope and control of the study. Achievement was not a measured outcome insofar as the children participating were not provided with scores on the outcome measure. Each child was always praised and reassured that they had done well regardless of their performance on the test. It was felt this was adequate in relation to this study as this reassurance meant that the children should not have any express negative or overly positive feelings regarding their achievement on this test. Of course, their own feelings about their achievement cannot be ruled out.

Personality and socio-economic status were also not accounted for, nor was religious preference. And overall, if all of these factors have influence over a child’s attitude then of course those attitudes will be a factor in how much they engage with mathematical learning. The vast majority of the children who took part in the study engaged well with all the stages of the research and appeared to have a good attitude to mathematics and learning, a very small number absolutely refused to engage, and for those children the data was incomplete, therefore as previously discussed they were eliminated from the final dataset (Chapter 4).

Considerable thought was given to the inclusion of a demographic data, but the schools found this unfavourable. There was a belief that parents would be less inclined to allow their children to participate if they felt that their home life, income and education levels were being recorded. While many studies have included socio-demographic data, a recent study by Navarro et al (2012) sought to ascertain and monitor early numerical competency in kindergarteners and first graders by means of using socio-economic variables alongside the Early Numeracy Test (ENT). In terms of the socio-demographic results their main finding was that the children whose homes had two or more computers were significantly more likely to be in the higher achieving numeracy group. Moreover they found no evidence for a specific socio-demographic profile unique to children in either a low or a high achieving group. In light of this relatively new evidence it is not likely a major flaw that socio-demographic data was not included in this thesis.

10.7 Recommendations for further study

In a theoretical sense there is scope for discussion and detailed examination of the fractionation of CE-CWM element of working memory in young children. These measures demonstrate the required correlational relationship to indicate that they belong to the same theoretical construct yet this study identifies a complex relationship between the separable aspects of CE-CWM (in terms of verbal and nonverbal working memory) and curricular mathematics whereby a developmental shift appears to be taking place. This could be explored further by modifying the design to include enough CE-CWM variables from the separable aspects to be able to conduct analyses that examines latent variables and construct structure.

In a related idea, earlier in this chapter the shift in the relationships between CE-CWM (from a nonverbal to a verbal measure) and mathematics was discussed. It is suggested that this might have some theoretical basis in the form of integration between V-STM and V-CWM and the episodic buffer. A very speculative idea is that at around the age of six some kind of integration is occurring between verbal short-term memory and verbal complex memory and this has been hinted at previously (Smith, 2005) although not fully examined. It is also possible that this has a bearing on how children perform on mathematical tasks. There are still some issues with this however; tasks claiming to measure the episodic buffer are still not easily identifiable, few studies claim to have measured the episodic buffer (see, for example Henry, 2010) and applied research with young children in this field is currently limited. It would therefore be important to develop reliable measurement tools for use with children before this could be put into action with regard to mathematics and even the curriculum.

Given the heavy current research focus on early numeracy (Navarro et al., 2012; Passolunghi et al., 2007; Passolunghi & Lanfranchi, 2012), the Approximate Number System (Bonny & Lourenco, 2013; Gilmore et al., 2011; Libertus et al., 2013; Mazzocco et al., 2011), non-symbolic and magnitude representation(De Smedt, Verschaffel, et al., 2009; Herrera, Macizo, & Semenza, 2008; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Soltész, Szűcs, & Szűcs, 2010) it would be expedient to extend the use of tests of early numeracy to children similar to those in this thesis and mapping this on to UK curricular mathematics.

This thesis has shown that working memory has an impact on curricular mathematics, in particular Calculation, and given the research by Holmes et al (2009) that indicates that adaptive working memory training provides gains in both working memory span and mathematics then many of the recommendations would centre around this idea. It would be useful to measure CE-CWM and Calculation at school entry and provide adaptive training for those children identified as having low working memory span scores, with appropriate control groups, following up post-test to discover if any gains were long-term effects, and if they significantly impacted upon scoring on Calculation.

Another aspect of working memory training that may be useful to research is if the gains in working memory skills transfer to non-cognitive skills, such as increases in self-esteem and classroom motivation and decrements in negative areas such as anxiety and poor classroom behaviour.

There is also abundant room for examination of curricular tests versus laboratory analogues of classroom activities. The Mathematics 5-14 test was adopted in this study as it was deemed the most appropriate for measuring curricular mathematics, however when faced with the strand "crossover" of many of the questions in the mathematics test, it would be very useful to design and test a UK curricular standardised mathematics assessment that minimises strand crossover and will span all the primary school years.

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Appendices

Appendix A

Number Strand Learning Outcomes

R	Y1	Y2
Observe number relationships and patterns in the environment and use these to derive facts	Derive and recall all pairs of numbers with a total of 10 and addition facts for totals to at least 5; work out the corresponding subtraction facts	Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100
Find one more or one less than a number from 1 to 10	Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple	Understand that halving is the inverse of doubling and derive and recall doubles of all numbers to 20, and the corresponding halves
Select two groups of objects to make a given total of objects	Recall the doubles of all numbers to at least 10	Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; recognise multiples of 2, 5 and 10
		Use knowledge of number facts and operations to estimate and check answers to calculations

c.f. (DfEE, n.d.-a)

Appendix B

Calculation Strand Learning Outcomes

R	Y1	Y2
Begin to relate addition to combining two groups of objects and subtraction to 'taking away'	Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number	Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number; use practical and informal written methods to add and subtract two-digit numbers
In practical activities and discussion begin to use the vocabulary involved in adding and subtracting	Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number	Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences
Count repeated groups of the same size	Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups	Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders
Share objects into equal groups and count how many in each group	Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences	Use the symbols +, -, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. ☐ ÷ 2 = 6, 30 - ☐ = 24)

c.f. (DfEE, n.d.-a)

Appendix C

Problem Solving Learning Outcomes

R	Y1	Y2
Use developing mathematical ideas and methods to solve practical problems	Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change'	Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence
Match sets of objects to numerals that represent the number of objects	Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem
Sort objects, making choices and justifying decisions	Answer a question by selecting and using suitable equipment, and sorting information, shapes or objects; display results using tables and pictures	Follow a line of enquiry; answer questions by choosing and using suitable equipment and selecting, organising and presenting information in lists, tables and simple diagrams
Talk about, recognise and recreate simple patterns	Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions	Describe patterns and relationships involving numbers or shapes, make predictions and test these with examples
Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices	Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures	Present solutions to puzzles and problems in an organised way; explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences

c.f. (DfEE, n.d.-a)

Appendix D

Measures, Shape and Space Learning Outcomes

Shape			Measures		
R	**Y1**	**Y2**	**R**	**Y1**	**Y2**
Use familiar objects and common shapes to create and recreate patterns and build models	Visualise and name common 2-D shapes and 3-D solids and describe their features; use them to make patterns, pictures and models	Visualise common 2-D shapes and 3-D solids; identify shapes from pictures of them in different positions and orientations; sort, make and describe shapes, referring to their properties	Use language such as 'greater', 'smaller', 'heavier' or 'lighter' to compare quantities	Estimate, measure, weigh and compare objects, choosing and using suitable uniform non-standard or standard units and measuring instruments (e.g. a lever balance, metre stick or measuring jug)	Estimate, compare and measure lengths, weights and capacities, choosing and using standard units (m, cm, kg, litre) and suitable measuring instruments
Use language such as 'circle' or 'bigger' to describe the shape and size of solids and flat shapes	Identify objects that turn about a point (e.g. scissors) or about a line (e.g. a door); recognise and make whole, half and quarter turns	Identify reflective symmetry in patterns and 2-D shapes and draw lines of symmetry in shapes	Use everyday language related to time; order and sequence familiar events and measure short periods of time	Use vocabulary related to time; order days of the week and months; read the time to the hour and half hour	Read the numbered divisions on a scale, and interpret the divisions between them (e.g. on a scale from 0 to 25 with intervals of 1 shown but only the divisions 0, 5, 10, 15 and 20 numbered); use a ruler to draw and measure lines to the nearest centimetre
Use everyday words to describe position	Visualise and use everyday language to describe the position of objects and direction and distance when moving them, for example when placing or moving objects on a game board	Follow and give instructions involving position, direction and movement			Use units of time (seconds, minutes, hours, days) and know the relationships between them; read the time to the quarter hour; identify time intervals, including those that cross the hour

		Recognise and use whole, half and quarter turns, both clockwise and anticlockwise; know that a right angle represents a quarter turn			

c.f. (DfEE, n.d.-a)

Appendix E

Items for Number Strand Questions (Mathematics 5, 6, 7) (NFER-Nelson, 2001)

Maths 5 (n =6)				
	Type	Question	Supported by images or props	Linked to other strand
1	Counting & Recognising Numbers	Which pair of hands shows nine?	Image of four pairs of hands each displaying a different number of fingers.	No
4	Counting & Recognising Numbers	This shows the numbers you can press on a telephone. There are three numbers missing. Write in all the missing numbers.	Image of a telephone number pad with missing numbers.	No
6	Counting & Recognising Numbers	Look at the box at the top. There are six dots in it. Fins another box which has the same number of dots. Put a tick inside the box.	One image with six dots (like a domino), and below it another four images with dots, only one showing six dots.	No
16	Counting & Recognising Numbers	Here are some shapes. Look at them all. Nazir made the shape with eight triangles. Put a tick on the shape Nazir made.	Image showing five shapes each comprised of a number of triangles.	No
18	Counting & Recognising Numbers	Which box has the most buttons in it?	Three boxes showing a small, medium and large quantity of buttons.	No
23	Counting & Recognising Numbers	Look at your shapes. Choose nine shapes.	Supported by card tiles made from a variety of shapes, in two colours. Child could choose any shape or any colour to the value of nine shapes.	No
Maths 6 (n = 6)				
5	Counting, properties of number & number sequences	Double each of the numbers and write your answers in the boxes.	No. Numbers displayed in a vertical list beside a box for the answer.	Calculation
11	Place value & ordering	Four people are standing in a queue to pay for their shopping. The boy is first in the queue. Who is third in the queue?	Yes. Images shows silhouette of a boy, girl woman and man, each labelled as such. Instructor could point to the boy.	Solving Problems
12	Place value & ordering	What number is 10 more than 7? Write your answer in the box.	No.	No
15	Place value & ordering	In the box write any number that is greater than 3 but less than 12.	No.	No
16	Counting, properties of number & number sequences	How many pairs of socks are there? Write your answer in the box.	Yes. Images shows six single socks. Child is expected to answer 3 pairs.	Calculation
20	Place value & ordering	Write these numbers in the boxes. Start with the smallest and end with the largest.	No. Series of numbers is presented in random order and random distribution on the page.	No
Maths 7 (n= 6)				
11	Place value & ordering	Five children are standing in a bus queue. Sophie is first in the queue. Who is fifth in the queue? Write down their name.	Yes. Image shows a queue of 5 people.	Solving Problems

13	Place value & ordering	Here are four numbers. Find the right name for each number.	No. Vertical list of four similar numbers (59, 48, 49, 58) and unordered vertical list of corresponding number names.	No
15	Place value & ordering	This sum has one coin missing. Which coin is it?	Yes. Images of a 10p, a 20p and a box to represent the missing coin. Sum should equal 50p, represented by a 50p coin.	Solving Problems
16	Fractions	One of these squares has one quarter coloured green. Decide which one it is and put a tick on it.	Yes. Four images divided into half, thirds, quarters and sixths, each with only one coloured fraction.	No
18	Place value & ordering	Write the correct numbers in the boxes.	No. 28 =___ tens + ___ ones 40 =___ tens + ___ ones	No
28	Place value & ordering Estimating & rounding	Which of these numbers is nearest to two hundred and fifty? Put a tick on it.	No. Vertical list of four numbers (350, 240, 300, 280).	No

Appendix F

Items from the Calculation Strand (mathematics 5, 6, 7) (NFER-Nelson, 2001)

Maths 5 (n =3)				
	Type	Question	Supported by images or props	Linked to other strand
3	Adding and Subtracting	Look at the first box. This shows that Tola had four balloons. Then she blew up three more. Draw all Tola's balloons in the next box.	Yes. Two boxes, one with four balloons and one empty box. Any representation of seven would be acceptable as an answer.	No
8	Adding and Subtracting	Which domino has seven dots altogether? Put a tick under it.	Yes. Five dominos each with a different number of dots.	No
15	Adding and Subtracting	There are five apples on the plate. Laura eats two apples. On the bottom plate draw the apples which are left.	Yes. Two plates, one with five apples on and one empty plate. Any representation of three would be acceptable as an answer.	No
Maths 6 (n = 13)				
1	Adding and Subtracting	Ten children were asked whether they liked tomatoes. The number of children who like tomatoes is shown inside the circle. How many children do NOT like tomatoes?	Yes. Circle containing 6 stick people, four stick people outside the circle.	Problem Solving
3	Adding and Subtracting	There are eight coins in your purse. You have three friends. You give each friend one coin. How many coins will be left in your purse?	Yes. Purse shape containing eight coins.	Problem Solving
5	Adding and Subtracting	Double each of the numbers in and write your answers in the boxes.	No. Numbers, 2, 3, 4 each with an arrow to the blank answer box.	Number
6	Adding and Subtracting	There are ten sweets in the bag. You give two sweets to your friend. How many do you have left?	Yes. Bag with ten sweets in it.	Problem Solving
7	Adding and Subtracting	This question says "what must be added to 3 to make seven?"	No.	No
12	Adding and Subtracting	What number is ten more than seven?	No.	Number
13	Adding and Subtracting	Here are the prices of three types of sweet: a mouse, a bootlace and a chew. You buy three sweets, one of each type. How much do they cost?	No. Prices listed adjacent to each sweet name.	Problem Solving
14	Adding and Subtracting	There are three apples in the basket. There are six apples in the tree. How many apples are there altogether?	Yes. Basket containing apples and a tree with apples in it.	Problem Solving
16	Adding and Subtracting	How many pairs of socks are there?	Yes. Six identical socks.	Number, Problem Solving
18	Adding and Subtracting	Work out the answer and write it in the box.	No. Sum 6-2.	No
19	Adding and Subtracting	How much do these coins add up to?	Yes. Image of a 10p coin, a 5p coin, two 2p coins and a penny.	Problem Solving
22	Adding and Subtracting	Find two numbers that add up to nine.	No. ☐+ ☐ = 9	Problem Solving
25	Adding and Subtracting	The question says "Sweets cost 4peach". Katie buys 2 sweets. How much does she spend?	No. Question is written.	Problem Solving
Maths 7 (n= 8)				
3	Adding and Subtracting	Find two odd numbers that add up to eight.	No. ☐ + ☐ = 8	Problem Solving
6	Adding and Subtracting	The question says "what must be added to 8 to make 17?"	No. Question is written.	Problem Solving

9	Multiplication and division facts	This question says “Apples cost 23 pence each. I buy three apples. How much do I spend?”	No. Question is written.	Problem Solving
15	Adding and Subtracting	This sum has one coin missing. Which coin is it?	Yes. Image of a 10p coin + 20p coin + ☐ = 50p coin	Number, Problem Solving
19	Adding and Subtracting	This question says “I have 20 pence. I buy a biscuit for 8 pence. How much money do I have left?	No. Question is written.	Problem Solving
21	Adding and Subtracting	Here are two number machines. The first one adds three to any number that you put in. What does the second machine do?	No. Machine One is a box with a 2 being fed into it via an arrow. In the centre of the box is +3 and another arrow leads to number 5. Machine Two omits the operation in the centre of the box.	No
23	Adding and Subtracting	Here are three sum and three answers that are mixed up. Find the right answer for the sum and draw an arrow to its right answer.	No. Question is written.	No
25	Adding and Subtracting	Find the missing numbers. Write them in the boxes.	No. ☐ - 4 =2 ☐ - 3 =2	No

Appendix G

Items from the Problem Solving Strand (Mathematics 5, 6, 7) (NFER-Nelson, 2001)

Maths 5 (n =5)				
	Type	Question	Supported by images or props	Linked to other strand
5	Comparing heights	The man and the lady want to buy a cupboard. The cupboard has to be shorter than the man but taller than the lady. Which cupboard is the only one they can buy?	Yes. Images of a short lady, a tall man and four cupboards	No
11	Ordering	This is a story about one hungry mouse. The mouse came along and ate up all the cheese. The pictures have got mixed up, which picture should come last?	Yes. Four images of a mouse in various stages of eating a piece of cheese, the last image showing the mouse and a few crumbs.	No
13	Money	Take all the coins out of your pot. Now choose the coin Which is worth the most. Put it back in your pot and show me.	Yes. Supported by a pot with six small denomination coins with 10p being the largest denomination.	No
14	Shopping	In this question you will have to choose more than one coin. You want to buy a pencil. It costs seven pence. Which coins make seven pence?	Yes. Coins on table in front of them. The child could make up a total of 7p via two means. 2 x 1p, 1 x 5p or 1 x 2p, 1x5p.	No
21	Sorting shapes	I will sort my shapes into two piles. Look at how I sort my shapes. (Instructors shapes are separated into two piles based upon the colour of the shape) Now sort all your shapes in the same way as I did.	Yes. A pile of black and orange mixed shapes.	No.
Maths 6 (n = 13)				
1	Making decisions & Organising and using data	Ten children were asked whether they like tomatoes. The number of children who like tomatoes is shown inside the circle. How many children do not like tomatoes?	Yes. Image shows six stick men in a circle and 4 stick men positioned outside the circle.	Calculation
3	Making decisions & Problems involving real life, money and measures	There are eight coins in your purses. You have three friends. You give each friend one coin. How many coins will be left in your purse?	Yes. Image of a purse shape containing 8 identical coins.	Calculation
6	Making decisions & Problems involving real life, money and measures	There are ten sweets in the bag. You give two sweets to your friend. How many do you have left?	Yes. Image of a shape containing 10 sweets.	Calculation
8	Reasoning about numbers and shapes	Squares and circles are drawn in a pattern. A group of squares is followed by a group of circles. How many squares are there in each group?	Yes. Pattern of 3 squares followed by 3 circles.	No
10	Organising and using data	This shows how fifteen families travelled on holiday. The words say Boat, Plane, Train and car. Add together the number of families who went by train and car.	Yes. Crude graph representing the data. The modes of travel were described by word and by picture.	No

11	Making decisions & Problems involving real life, money and measures	Four people are standing in a queue to pay for their shopping. The boy is first in the queue (point to boy if necessary). Who is third in the queue?	Yes. Shadow images of a boy, girl, woman and man in a queue, each with a shopping trolley.	Number
13	Making decisions & Problems involving real life, money and measures	Here are the prices of three types of sweet: a mouse, a bootlace and a chew. You buy three sweets – one of each type. How much do they cost altogether?	No. List of sweets, saying mouse 1p, bootlaces 2p, chew 4p	Calculation
14	Making decisions	There are three apples in the basket. There are six apples in the tree. How many apples are there altogether?	Yes. A Tree with 6 apples depicted, and 3 in a basket.	Calculation
16	Problems involving real life, money and measures	How many pairs of socks are there?	Yes. Image of 6 socks, equalling 3 pairs.	Calculation
19	Problems involving real life, money and measures	How much do all these coins add up to?	Yes. Image of 1 x 10p, 2 x 2p, 1 x 5p, 1x 1p	Calculation
22	Reasoning about numbers and shapes	Find two numbers that add up to nine.	No. ☐ + ☐ =9	Calculation
25	Making decisions	The question says, sweets cost 4 pence each. Katie buys 2 sweets. How much does she spend?	No. The question is written down.	Calculation
26	Problems involving real life, money and measures	What time does the clock show?	Yes. A clock shows 3 o'clock	No
Maths 7 (n= 16)				
3	Reasoning about numbers and shapes	Find two odd numbers that add up to 8. Remember both numbers must be odd.		Calculation
4	Problems involving real life, money and measures	Which clock says half past eight?	Yes. Three clocks showing three different times.	Measures, Shape and Space
5	Organising and using data	This shows a way of sorting animals. A snail has a shell and no legs. Put a tick to show where a snail would go.	Yes. Grid with rows for legs, no legs and columns for shell, no shell accompanied by a drawing of a snail.	
6	Reasoning about numbers and shapes	The question says, what must be added to 8 to make 17?	No. The question is written down.	Calculation
7	Reasoning about numbers and shapes	This is a number pattern. Write down the next three numbers in the boxes.	No. Pattern is as follows 4, 4, 3, 4, 4, 3, 4 __, __, __	
8	Organising and using data	This shows which children own which pet. Who has two pets? Write down the name of the child.	Yes. Image shows list of three names and pictures of three animals. There are lines indicating which child owns which pet, and only one child has two lines extending from their name.	

9	Making decisions, Problems involving real life, money and measures	This question says “Apples cost 23p each. I buy three apples. How much do I spend?”	No. The question is written down.	Calculation
11	Making decisions, Problems involving real life, money and measures	Five children are standing in a bus queue. Sophie is first in the queue. Who is fifth in the queue? Write down their name.	Yes. Drawing of five children in a queue.	Number
14	Organising and using data	A group of children were asked which fruit they liked best. This shows how many children chose each fruit. Which fruit do most children like? Write your answer on line A. How many more children like bananas than plums? Write your answer on line B	Yes. Supported by a pictorial graph representing fruit (both in name and by picture) on the x axis, and number of children on y axis.	
15	Problems involving real life, money and measures	The sum has one coin missing. Which coin is it? Write the value of the coin in the box.	Yes. Image shows 10p + 20p + ☐= 50p depicted by drawings of the coins.	Number, Calculation
19	Making decisions, Problems involving real life, money and measures	This question says I have 20 pence, I buy a biscuit for 8 pence. How much money do I have left?	No. The question is written down.	Calculation
20	Organising and using data	This shows how many cars passed the school gate in one minute. The cars were blue, red, black or white. Each cross stands for one car. Another red car goes by. Show this on the chart.	No. Displayed as below Blue cars x x Red cars x x x x Black cars x x x White cars x x x x x	
22	Organising and using data	This is a price list for beef burgers, oven chips, fish fingers and peas. For each food type there are two prices: one for a small pack and one for a large pack. Complete the sentence below the table. A small pack of fish fingers costs….	No. A price list is displayed as described, and the question is written below it.	
24	Organising and using data	Four children had a race round the playground. This shows how long each one took. Who finished first?	No. A bar chart with name of child on x axis and number of minutes on y axis.	
26	Problems involving real life, money and measures	Look at this balance. Is the parcel heavier or lighter than the sand? Or is it the same weight? Put a tick on the right answer.	Yes. A set of balance scales with sand on the left, and a heavy parcel on the right.	
27	Problems involving real life, money and measures	This shows a way of sorting shapes, but one shape is in the wrong place. Some shapes have a straight edge and some shapes have a curved edge. What shape is in the wrong place?	Yes. A venn diagram showing straight edged shapes on one side, curved on the other, a half-moon shape in the centre of the diagram, and a straight edged shape in the incorrect position.	Measures, Shape and Space

Appendix H

Items from the Measures, Shape and Space Strand (Mathematics 5, 6, 7) (NFER-Nelson, 2001)

Maths 5 (n =10)				
	Type	Question	Supported by images or props	Linked to other strand
2	Clocks	Look at these clocks. One of them shows four o'clock. Put a tick on it.	Yes. Four clocks showing four different times.	No
7	Reasoning about shapes	Which T Shirts have white flowers? Put a ring round each t-shirt that has white flowers.	Yes. Eight images of t-shirts with two that have white flowers.	No
9	Quantity	Rachel has a mug if juice. She drinks some of her juice. The mug is now half full Draw a line to show where the juice will come up to when the mug is half full.	Yes. An image of a mug.	No
10	Recognising Shapes	Find all the triangles in the box. Put a tick inside each triangle.	Yes. A box containing 12 random shapes, including five triangles of differing types.	No
12	Comparing Shapes	Ryan has some different shaped tiles in a box. He has taken one out (point to a tile outside the box) Now look at the tiles in the box. Which tiles are the same shape as the one which Ryan has taken out. Put a ring around each of them.	Yes. One square tile outside the box. A box with eight shapes including three squares at different rotations.	No.
17	Weighing	Look at the teddies and the presents. They are on balances. One of the presents is heavier than the teddy. Which present is it? Put a tick on it.	Yes. Three separate images of a teddy and a present on a balance scale. One the same weight, one lighter and one heavier.	No
19	Repeating Patterns	Look at the shapes and colours in this repeating pattern. Copy the pattern with your shapes then carry on the pattern.	Yes. Show a printed pattern of black and orange shapes in a repeating pattern of two cycles. The child should copy the pattern and repeat for two cycles using corresponding tiles. Can prompt to repeat a second cycle.	No
20	Copying Patterns	Look at the shapes and colours in this pattern. Use your shapes to copy the pattern.	Yes. Show a printed pattern and child should replicate the pattern.	No
22	Describing Shapes	Look at your shapes (show a rectangle) Choose a shape which is exactly the same as this one. Show me a shape which has four sides which are all the same length. Show me a shape which has three sides which are all different lengths.	Yes. Supported by a series of tiles.	No
24	Counting Shapes	Look at your shapes choose five squares. Show me.	Yes, Supported by tiles of varying shape and colour. Child could choose five squares of any colour.	No.
Maths 6 (n = 7)				
2	Shape and Space	Tick the shape which is below the square.	Yes. Five shapes with a square in the centre.	No
4	Shape and Space	One of these shapes has four corners that are the same. Put a tick on it.	Yes. Four shapes each with four sides. Child should choose the square.	No
9	Shape and Space	One of these shapes does not have three sides. Put a tick on it.	Yes. Five shapes, only one is not a triangle.	No

21	Shape and Space	One of these circles has one half coloured. Put a tick on it.	Yes. Four circles shaded to different degrees.	No
23	Measures	Which is the tallest tree? Put a tick in the box below it. Which is the shortest tree? Put a cross in the box below it.	Yes. Image of four different trees of varying sizes.	No
24	Shape and Space	One of these shapes has curved sides and straight sides. Put a tick on it.	Yes. Three regular shapes and two irregular shapes.	No
26	Measures	What time does the clock show? Write your answer in the space.	Yes. Image of a clock showing 3 o'clock.	Problem Solving
Maths 7 (n= 8)				
1	Measures	Which is the longest straw? Put a tick on it. Which is the shortest straw? Put a ring around it.	Yes. Image of four straws of varying lengths.	No
2	Shape and Space	This table shows where four children are sitting round a table. Who is sitting opposite Ben?	Yes. Square with four circles, one at each side with a child's name in it.	No
4	Measures	Which clock says half past eight? Put a tick on it	Yes. Three clocks with different times on.	Problem Solving
10	Shape and Space	What shape is a tin of beans? Put a tick on the right name.	Yes. Image of a tin of beans and four shape names.	No
12	Shape and Space	The first box shows a letter T the right way up. In the second box it has turned through one right angle. It is about to turn through one more right angle in the same direction. What will it look like? Draw it in the empty box.	Yes. Three boxes, T right way up, T rotated one right angle and a blank box.	No
17	Shape and Space	Look at the names in the boxes and the shapes below. Find the right name for each shape and write is down.	Yes. Four shape names and three shapes.	No
26	Measures	Look at this balance. Is the parcel heavier or lighter than the sand? Or is it the same weight. Put a tick on the box with the right answer.	Yes. Scales show a parcel is heavier than the sand, child should identify the word that corresponds.	No
27	Shape and Space	This shows a way of sorting shapes, but one shape is in the wrong place. Some shapes have a straight edge and some shapes have a curved edge. What shape is in the wrong place?	Yes. A venn diagram showing straight edged shapes on the left, curved edge shapes on the right. A half circle in the intersect, and a straight edged shape in the curved side.	Problem Solving